Georg Hajdu

# Syntactic considerations on the transcription of and modulation between microtonal scales

## Opening remarks

Since the tenures of György Ligeti and Manfred Stahnke, the Hamburg University of Music and Drama (HfMT) has been involved in the study of microtonal music. Continuing in this tradition, I have founded in 2010 the Center for Microtonal Music and Multimedia (ZM4). The ZM4 is focussed on the creation of new instruments, the study of novel notations and approaches to the theory and practice of microtonality and specifically of the Bohlen-Pierce scale. In this context, the Bohlen-Pierce Clarinet project was started in 2008 and led within the 13 years of its existence to the premiere of numerous compositions and two doctoral projects by Nora-Louise Müller and Todd Harrop as well as three different book projects. A CD with Bohlen-Pierce compositions written by composers associated with the HfMT has been released in 2020 by Genuin[1]. A collaborative full-evening theatrical project with the renowned observatory in Hamburg-Bergedorf is planned for 2022 and is supposed to feature a large ensemble of Bohlen-Pierce instruments.

## Introduction

This paper is about syntactical considerations in the context of the modulation between microtonal scales, particularly between the "odd-number world" of the Bohlen-Pierce scale and the "even-odd world" of the 12-tone chromatic and diatonic scales[2]. We are following the well-established assumption that the basic notion of tension and release as well as the principles of gestalt psychology give birth to complex hierarchical structures governing musical syntax. Ground-breaking studies have been undertaken by Lerdahl and Jackendoff ("generative grammar") as well as Krumhansl ("a cognitive model of tonality") to further the understanding of hierarchy in music [1][2]. Barlow developed a computational model centered on harmonic and metric profiles which serves as the basis for much of my work, including the software DJster [3]. In 1989, I have extended his notion of *harmonicity* by introducing the concept of harmonic energy. In this paper, I will show that when creating a generative process which performs a modulation between the scales, tonality profiles may play a crucial role.

## Cognitive model of tonality and key areas

In 1990, Carol Krumhansl published her book *Cognitive Foundations of Musical Pitch* describing the results of a series of experiments with the *Probe Tone Method*. In this book, Krumhansl described the construction of a toroidal key space by performing multi-dimensional scaling on the results of the aforementioned method. David Wessel suggested that this model could also be used to generative music simply by simulating movement along the surface of the torus[3]. Inspired by Wessel's idea, I created a Max patch in 1992 in which an artificial neural

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[1] https://www.genuin.de/de/04_d.php?k=564
[2] The term "world" refers to the mathematical foundations of these particular scales in their repercussions in terms of spectral and intervallic qualities.
[3] Personal communication

network performs an interpolation between profiles in response to movement over a 2D map of the torus. The music is created by a probabilistic process operating on those profiles, i.e. the average rating for a pitch class determines the probability of a pitch being heard.

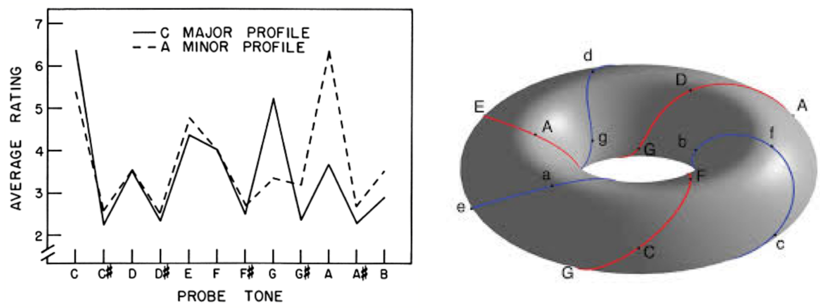

Figure 1: Tonality profiles for C major and c minor (left) and the key space derived by multidimensional scaling

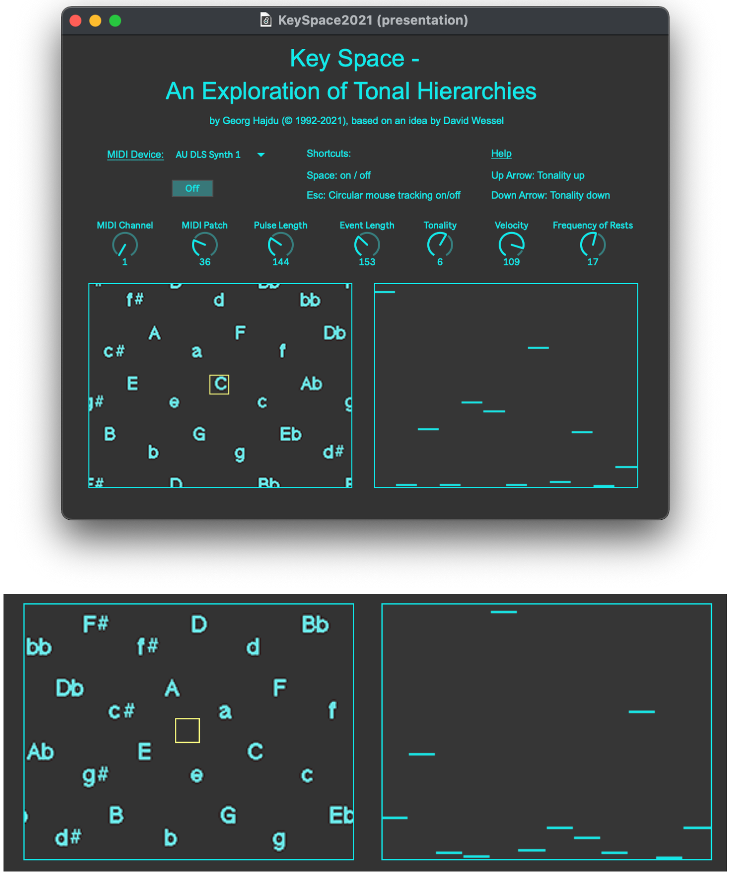

Figure 2: Screenshot of the Key Space application (top) and an interpolated state between four different key areas (bottom)

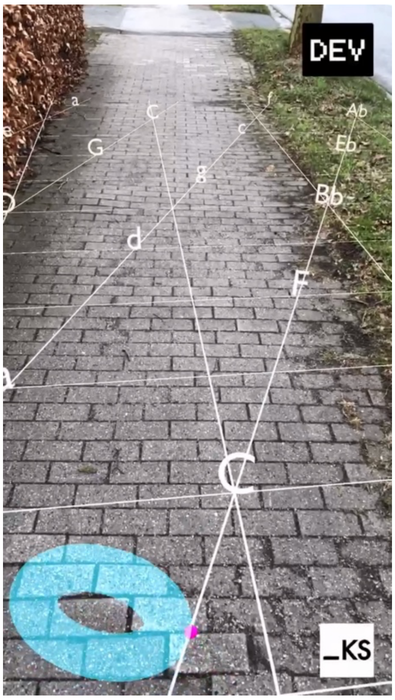


*Figure 3: Photo of an augmented reality application currently in development by Sebastian Olariu of the Hamburg-based MMKH, based on the Max Key Space patch. Note the turquois-colored torus with the purple dot indicating the path a user takes on its surface.*

## Mathematical model of tonality

In the 1970s, Clarence Barlow developed a quantitative approach to tonality and meter in the context of his piano piece *Çoğluotobüsişletmesi*, which he also applied to his real-time event generator AUTOBUSK. His algorithms are based on following principles and their shrewd combination [4]:

- Metric hierarchy: Indispensability of a pulse for a given meter, resulting in a metric profile
- Tonal hierarchy: Indigestibility of numbers determining the harmonicity of an interval, resulting in a tonal profile

## Harmonic energy

As Barlow's harmonicity formula yields discrete values for any given frequency ratio, I derived the concept of a continuous harmonic-energy curve by inverting the harmonicity values and applying a bell curve with a given width around each interval (Figure 4) [5]. The depth and width of resulting troughs thus serve as a stability indicator. This approach also shows why most mathematically possible intervals are in practice eclipsed by strong intervals in their vicinity, just like fainter stars in the neighborhood of brighter ones. However, lowering the width of a bell curve gives fainter intervals a chance to reveal themselves (Figure 4 middle). These are for the most part theoretical considerations as it depends on many factors as to how much a listener (musician or non-musician) is capable of differentiating between (adjacent and non-adjacent) microtones or to what extent an instrument is even capable of representing the wealth of intervallic possibilities.[4]

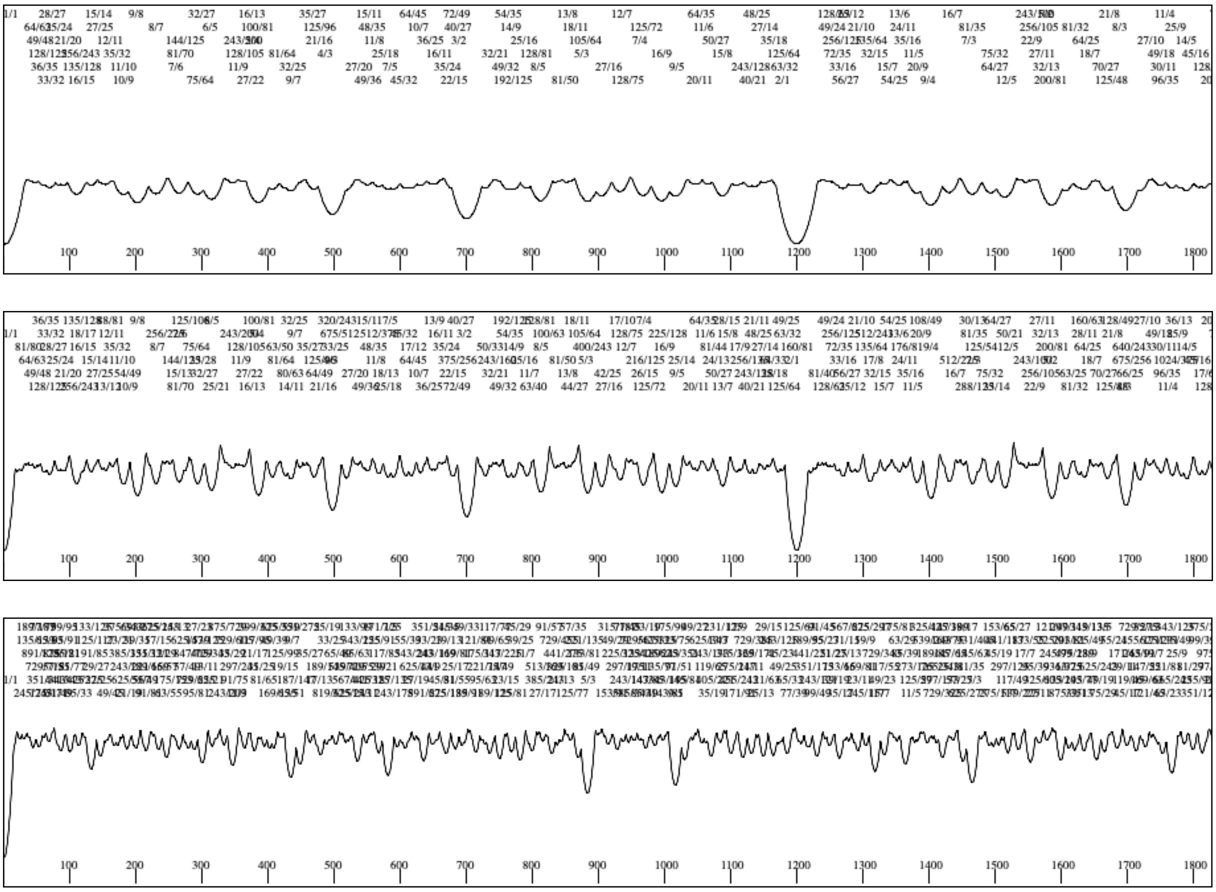


*Figure 4: Harmonic energy curves can serve as lookup tables to yield a ratio for a specific interval. Curves are shown for odd and even intervals (width = 18; top), odd and even intervals (width = 10; middle), odd intervals (width = 8; bottom).*

Theorical considerations also apply to my work with the Bohlen-Pierce scale which is derived from odd frequency ratios. I, therefore, created a variant energy curve which does not include even numbers (Figure 4 bottom). A comparison between the middle and bottom curves in demonstrates that a large number of common intervals such as the octave are absent as they made up of ratios containing even numbers[5].

The reader might wonder about validity of such an approach, that ignores the octave, an interval that appears to be hardwired into our nervous system. However, there are fundamental acoustic phenomena that exist without out it, such as the triangle and square waves or the tones of the lower octave of a clarinet. From a composer's perspective, it does make a conceptual difference, though, whether to consider the interval of 1200 ¢ as a stable 2/1 or being in the vicinity of a rather feeble 351/175. Such concepts guide the creative mind to conceive of objects in abstract musical space that follow a logic merely seemingly at odds with the tenets of music perception.

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[4] See Philipp Gerschlauer's rendering of 128 notes per octave on an alto saxophone https://www.youtube.com/watch?v=lGa66qHzKME for an impressive example in the context of a wind instrument.

[5] The odd-ratio curve contains more yet less stable intervals and appears flatter than the odd-even curve – a finding which is related to *spectral flatness* – a term that refers to the noisiness of a spectrum.

## Bohlen-Pierce scale

The Bohlen-Pierce scale was invented (some would rather use the term *discovered*) independently by three people in the 1970s and 80s [6][7][8]. No other non-octave scale has been studied more thoroughly. It divides the 3/1 ratio (tritave = 1902¢) into 13 steps and its intervals are made up of ratios consisting of the numbers 3, 5, 7 and their powers only. It can therefore be defined as 7-limit complement-2 tuning, the equal-tempered version differing from its just version to a much lesser degree than 12ED2 from the 5-limit 12-tone scale.

## Notation of the BP scale

Working with a novel non-octave scale necessitates the creation of a new framework for music notation in which the following considerations will have to be taken into account:

- An ensemble for microtonal music typically consists of a mix of different instruments, some specifically built for the tuning, some adapted to it and some standard instruments such as string instruments.
- Members of the ensemble may require different approaches to reading music.
- Three types of notation exist and may be applied in rehearsal and concerts: conventional, logical and instrumental.

Together with Nora-Louise Müller, I devised a new notation system for the Bohlen-Pierce scale and worked on an editor for MaxScore capable of switching between different views of a microtonal scale in real time; Figure 5 features alternate representations of the Bohlen-Pierce scale [9] notated in MaxScore.


*Figure 5: Alternative notations of the middle tritave of the Bohlen-Pierce scale.*

## Bohlen-Pierce Microtonality

As the Bohlen-Pierce scale (13ED3) has an approximate step size of three quartertones, it is often labelled as a macrotonal scale. Two subdivisions of the BP step have gained increasing popularity: The division into three and five smaller steps have been referred to as BP triple

(39ED3) and quintuple scales (65ED3). Müller and Hajdu proposed the following set of accidentals taken from the Standard Music Font Layout to differentiate between steps[6]:

- 39ED3: Xenakis one and two third-tone sharps for upward motion and Wyschnegradsky two and eight twelfths-tone flats for downward motion
- 65ED3: A subset of the Spartan Sagittal single-shaft and multi-shaft accidentals for upward and downward motion

*Figure 6: Bohlen-Pierce triple (left) and quintuple scales (right)*

## In-between worlds

I will now present three scenarios in which transitions between the odd-even world of 12ED2 and the odd world of 13ED3 and their subdivisions play a role. We will first look into a *transcription* of a piano piece by Alexander Scriabin into 12-tone equal temperament, then turn our attention to *hybrid* pieces combining both scales and finally present an approach which can be the basis for generative processes *modulating* between BP and the chromatic and diatonic subsets of 41ED2.

### *Transcription of Vers la flamme by Alexander Scriabin*

*Vers la flamme* is one of Alexander Scriabin's last works. His oeuvre is characterized by the search of a new tonality in which the traditional polarity of tonic and dominant is increasingly replaced by the tritone, its subdivision into minor thirds and the octatonic scale. Treating the tritone as consonance impacts the sequence of tension and release and thus the syntactical aspects of his music. Scriabin's later works tend to be rather static, yet his work *Vers la flamme* represents an "evolutionary" step in that it seems to be moving relentlessly towards a harmonic goal, the ultimate release of tension, manifesting itself in the final arpeggiated chord reminiscent of a harmonic spectrum. From his communications with Leonid Sabaneyev, it is safe to assume that much of Scriabin's composition embodies ideas existing in "abstract" harmonic and melodic space. And this abstract harmonic space affords a number of "transcodings" of which the ones in 12EDO and 13EDT are merely two possibilities.

Scriabin's preoccupation with the tritone as a consonance, foreshadows the discovery of the Bohlen-Pierce scale. And with this in mind I attempted to transcribe the piano piece, which

[6] SMuFL, https://w3c.github.io/smufl/

has fascinated me since my youth, for an ensemble of Bohlen-Pierce instruments [10]. A recording of it is available on the CD *Beyond the Horizon*.

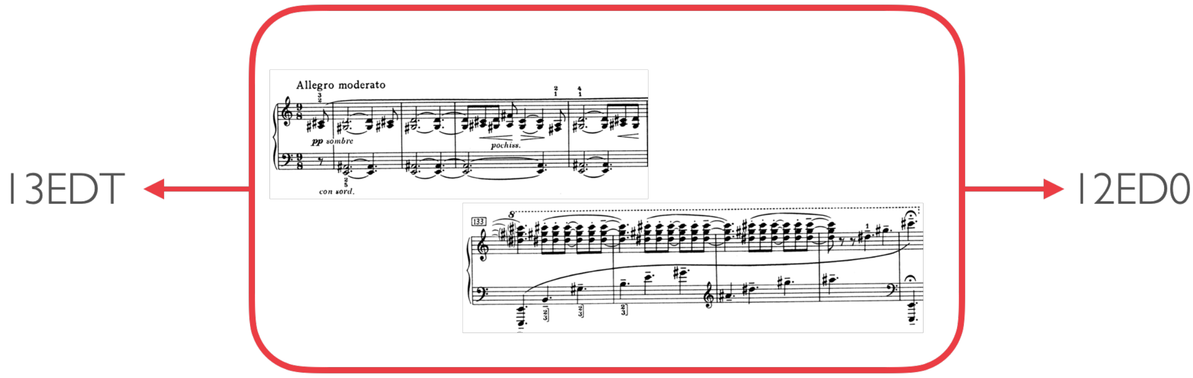


*Figure 7: Compositional Ideas are sometimes developed in abstract harmonic space and adjusted to a particular tuning for practicality's sake.*


*Figure 8: An ensemble of Bohlen-Pierce instruments performing Burning Petrol, a transcription of Scriabin's piano piece Vers la flame by Georg Hajdu.*

### Combined Scale (Harrop, 2014)

The CD *Beyond the Horizon* also includes two compositions calling for the combination of a Bb and a Bohlen-Pierce clarinet resulting in what we may call poly-microtonality. While in Fredrik Schwenk's piece *Night Hawks*, the two clarinets "circle" each other in constant imitation, Sascha Lemke, in his composition *Pas de Deux*, is making a deliberate attempt to interleave the pitches of two clarinets to create a 31-tone[7] symmetrical mode from a 247-tone[8] superset. Lemke chose the title Pas de Deux to represent this very idea: *Pas* in French

[7] 31 = 13 + 19 - 1
[8] 247 = 13 x 19

not only means *step* but also *not*; the title therefore also reads “not of two”. In measures 49 – 56, the two clarinets are interlocking to realize this particular 31-tone mode. In [11], Harrop included a table detailing this mode, referring to the basic interval of the 247-tone superset as bù and explained how he derived the unit—an approach which he revised after the publication of his article:

> *I’ve since updated the bù unit... Instead of the 247th root of 3 or 2.998 as I wrote in that book, it’s now the 362nd root of 5 (which amounts to nearly the same thing). If we’re going to split hairs, then 362ed5 has the advantage of splitting any error evenly over the octave and the tritave, meaning neither 2/1 nor 3/1 is favoured over the other. More importantly, working out the number of steps to 7/1 is more accurate with 362ed5, whereas 247ed3 would eventually lead to a rounding error in some ratio. So: 1 bù equals the 362nd root of 5/1, approximately 7.697 cents or 5.261 hekts, giving the following number of steps for primes 2, 3, 5 and 7: 156, 247, 362 and 438*[9].


*Figure 9: Night Hawks by Fredrik Schwenk. The second staff is in Bohlen-Pierce Soprano clarinet fingering notation (see Figure 5)*

[9] Personal communication on March 18, 2021


*Figure 10: Pas des Deux by Sascha Lemke. This passage shows how the clarinets interlock to perform a 31-tone symmetrical combination mode.*

| Pitch (name) | Interval (cents) | Interval (hekts) | Interval (bù) | Difference (bù) | Interval (1/8ths) | Difference (1/8ths) |
|---|---|---|---|---|---|---|
| A4, N | 0 | 0 | 0 | *n/a* | 0 | *n/a* |
| A#4 | 100 | 68 | 13 | 13 | 4 | 4 |
| O | 146 | 100 | 19 | 6 | 6 | 2 |
| B4 | 200 | 137 | 26 | 7 | 8 | 2 |
| P | 293 | 200 | 38 | 12 | 12 | 4 |
| C5 | 300 | 205 | 39 | 1 | 12 | 0 |
| C#5 | 400 | 273 | 52 | 13 | 16 | 4 |
| Q | 439 | 300 | 57 | 5 | 18 | 2 |
| D5 | 500 | 342 | 65 | 8 | 20 | 2 |
| R | 585 | 400 | 76 | 11 | 23 | 3 |
| D#5 | 600 | 410 | 78 | 2 | 24 | 1 |
| E5 | 700 | 478 | 91 | 13 | 28 | 4 |
| S | 732 | 500 | 95 | 4 | 29 | 1 |
| F5 | 800 | 547 | 104 | 9 | 32 | 3 |
| T | 878 | 600 | 114 | 10 | 35 | 3 |
| F#5 | 900 | 615 | 117 | 3 | 36 | 1 |
| G5 | 1000 | 684 | 130 | 13 | 40 | 4 |
| U | 1024 | 700 | 133 | 3 | 41 | 1 |
| G#5 | 1100 | 752 | 143 | 10 | 44 | 3 |
| V | 1170 | 800 | 152 | 9 | 47 | 3 |
| A5 | 1200 | 820 | 156 | 4 | 48 | 1 |
| A#5 | 1300 | 889 | 169 | 13 | 52 | 4 |
| W | 1317 | 900 | 171 | 2 | 53 | 1 |
| B5 | 1400 | 957 | 182 | 11 | 56 | 3 |
| X | 1463 | 1000 | 190 | 8 | 58 | 2 |
| C6 | 1500 | 1025 | 195 | 5 | 60 | 2 |
| C#6 | 1600 | 1094 | 208 | 13 | 64 | 4 |
| Y | 1609 | 1100 | 209 | 1 | 64 | 0 |
| D6 | 1700 | 1162 | 221 | 12 | 68 | 4 |
| Z | 1756 | 1200 | 228 | 7 | 70 | 2 |
| D#6 | 1800 | 1230 | 234 | 6 | 72 | 2 |
| E6 | 1900 | 1299 | 247 | 13 | 76 | 4 |
| N' | 1902 | 1300 | 247 | 0 | 76 | 0 |

*Table 1: Table created by Harrop detailing the mode used by Sascha Lemke. Reprinted with the kind permission of the author.*

In this context, I should also mention two other poly-microtonal compositions most notably *Bird of Janus* by Todd Harrop [12] (mixing the Bohlen-Pierce and Carlos Alpha scales) and *The Glacier*, a multimedia opera by Christian Klinkenberg in which Klinkenberg uses and occasionally interleaves four different microtonal scales, Bohlen-Pierce and 19ED2 among them [13].

## Modulation

The approach taken by Harrop is valid when looking for a method of creating a superset comprising both the Bohlen-Pierce and chromatic scales with ultimate precision, but such precision may not be always wanted or needed. The fact that the subdivision of the tritave (which serves as a BP ersatz octave) into 65 steps (i.e. the BP step into 5 intervals) yields (nearly) the same interval set as the division of the octave into 41ED2 is to be considered a serendipitous finding, enabling pivoting from ED3 to ED2 without having to resort to such a fine-grained structure as 247ED3.
However, while the pitches seem the same in terms of their frequencies, they occupy vastly different harmonic spaces. Therefore, pivoting between 65ED3 and 41ED2 is also a matter of interpolating between harmonic profiles.

| Step | Cents | 12ED2 | 13ED3 |
|---|---|---|---|
| 0 | 0 | D | N |
| 1 | 29 | | |
| 2 | 59 | | |
| 3 | 88 | Eb | |
| 4 | 117 | | |
| 5 | 146 | | O |
| 6 | 176 | | |
| 7 | 205 | E | |
| 8 | 234 | | |
| 9 | 263 | | |
| 10 | 293 | F | P |
| 11 | 322 | | |
| 12 | 351 | | |
| 13 | 380 | | |
| 14 | 410 | F# | |
| 15 | 439 | | Q |
| 16 | 468 | | |
| 17 | 497 | G | |
| 18 | 527 | | |
| 19 | 556 | | |
| 20 | 585 | G# | R |
| 21 | 614 | (Ab) | |
| 22 | 644 | | |
| 23 | 673 | | |
| 24 | 702 | A | |
| 25 | 732 | | S |
| 26 | 761 | | |
| 27 | 790 | Bb | |
| 28 | 819 | | |
| 29 | 849 | | |
| 30 | 878 | | T |
| 31 | 907 | B | |
| 32 | 936 | | |

| Step | Cents | 12ED2 | 13ED3 |
|---|---|---|---|
| 33 | 966 | | |
| 34 | 995 | C | |
| 35 | 1024 | | U |
| 36 | 1053 | | |
| 37 | 1083 | | |
| 38 | 1112 | Db | |
| 39 | 1141 | | |
| 40 | 1170 | | V |
| 41 | 1200 | D | |
| 42 | 1229 | | |
| 43 | 1258 | | |
| 44 | 1288 | Eb | |
| 45 | 1317 | | W |
| 46 | 1346 | | |
| 47 | 1375 | | |
| 48 | 1405 | E | |
| 49 | 1434 | | |
| 50 | 1463 | | X |
| 51 | 1492 | F | |
| 52 | 1522 | | |
| 53 | 1551 | | |
| 54 | 1580 | | |
| 55 | 1609 | F# | Y |
| 56 | 1639 | | |
| 57 | 1668 | | |
| 58 | 1697 | G | |
| 59 | 1726 | | |
| 60 | 1756 | | Z |
| 61 | 1785 | (G#) | |
| 62 | 1814 | Ab | |
| 63 | 1843 | | |
| 64 | 1873 | | |
| 65 | 1902 | A | N |

*Table 2: Alternative approach to combining the 12ED2 and 13ED3. 41ED2 provides an acceptable approximation to the equal tempered 12-tone scale.*

The maximum deviation between 12ED2 and the 12-tone subset of 41ED2 (12/41ED2, read 12 out of 41ED2) is 1200 / (41 * 2) = 14.6 ¢, thus about the same size as the deviation of the equal-tempered major third from 5/4. Considering that in musical practice deviations of up to 20 ¢ are quite common, the use of the 12-tone subset of 41ED2 presents itself as a viable compromise.

Figure 11 shows the modulation path between 13ED3 and 7/41ED2 (read 7 out of 41ED2) with 65ED3 and 41ED2 acting as pivot. The profile is derived from the harmonic energy curve (Figure 11) and show stable intervals such as unison, octave and tritave on top and unstable intervals on the bottom (the profile of the BP scale drawn in top left square of Figure 11). In the top center the increased number of intervals represents the 65-tone set of the quintuple scale. Pivoting to 41ED2 (top right) keeps the set size intact but changes the profile as we are now considering ratios also made up of even numbers (which noticeably alters the stability profile). Note how the octave (blue line) suddenly comes up from the "underground" and assumes its role as the interval for it is known. The second row represents the 12-tone (bottom left) and 7-tone, diatonic (bottom right) subsets created by increasingly removing pitches from the previous sets.
From a compositional perspective, it is obvious that each profile would mandate a different handling of the musical material via a distinct musical syntax. Particularly in the pivot phase, with many more intervallic choices, an approach emphasizing the equivalence of the intervals recommends itself—a tonally blurred phase before a new tonality "emerges from the ashes".

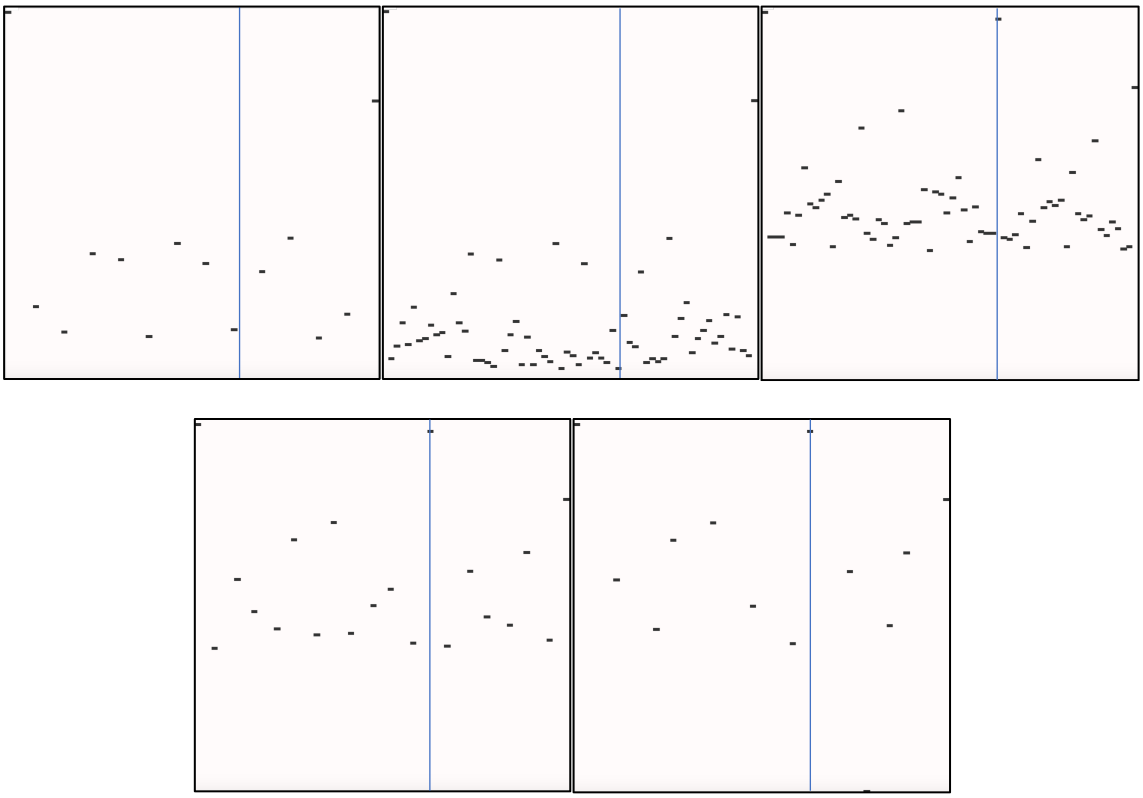

*Figure 11: Tonality profiles for the modulation path between 13ED3 and 7/41ED2. The direction is left to right, top to bottom. Each square represents a (sub)set of 65 intervals per tritave with interval size on the x axis and "harmonicity" on the y axis: the blue vertical line denoting the octave.*

## DJster

DJster is a generative music application capable of performing such a modulation path either in terms of an asynchronous process providing the opportunity to consider different alternatives between different "runs" of the program, or in terms of a synchronous event generation in an installation setting. DJster is a further development of Barlow's real-time event generator AUTOBUSK, which he originally developed for the Atari platform [3]. AUTOBUSK required that the tonal and metric profiles had to be pre-calculated for any given scale or meter. And this is where DJster fundamentally differs from its predecessor: Instead of pre-calculating the profiles, DJster performs a lookup in real time for a given scale. For instance, if a scale contained an interval of 386¢, the lookup would yield a just major third 5/4 in the even-odd ratio world but a rather exotic 81/65 in the odd ratio world with a substantially lower probability to occur than some of its other intervals.

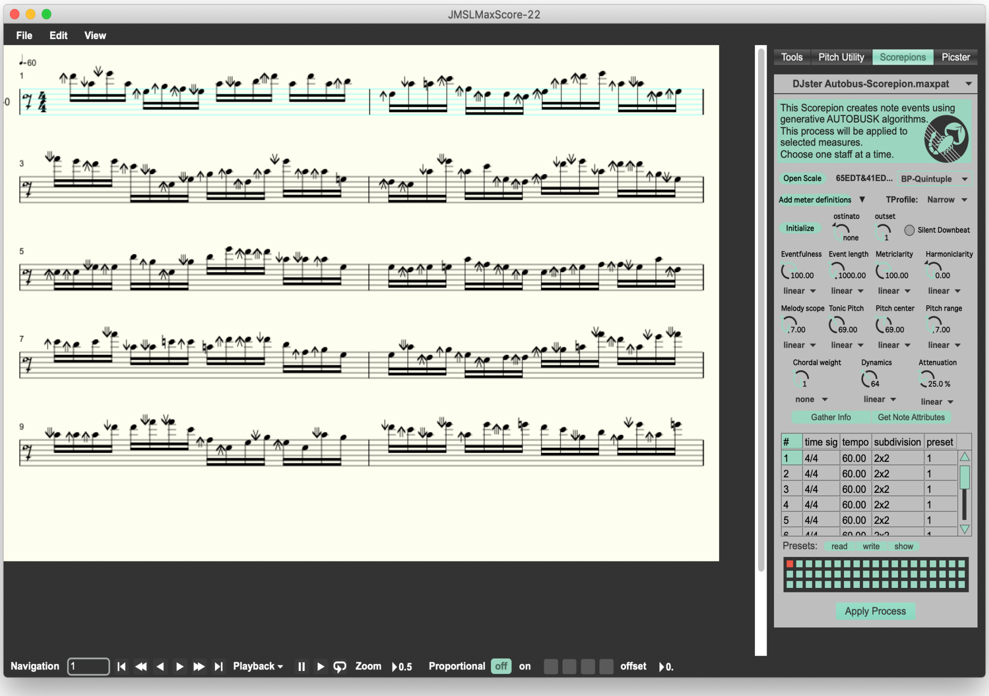


*Figure 12: Screenshot of DJster running as a plugin inside MaxScore. The music notated in the Bohlen-Pierce T clef represents the parameter setting visible in the sidebar. Note that the accidentals correspond to the set given in Figure 6 for the BP quintuple scale.*

## Conclusion

The transcription or modulation from one tuning to another, as well as the combination of tunings in terms of creating poly-microtonality and/or microtonal supersets is far from trivial. In this paper, we have seen that tonality profiles whether derived from experiments (such as the probe-tone method) or established through mathematical reasoning (such as the theories developed by Barlow, Tenney, Erlich and others) can serve as guides through musical territories, particularly when the compositional process is generative. Composers have to be keenly aware of the affordances of a particular tuning, as this has a fundamental impact on the syntax of the resulting music. This is particularly obvious for the odd-world of the Bohlen-Pierce scale where parallel sixths may need to be avoided and symmetries are built around the tritave instead of the octave. Finding a path between worlds is a challenge which composers such as myself would like to meet not just in terms of theory, but also in terms of actual future compositions.

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