Starting Over – Chances Afforded by a New Scale

Nora-Louise Müller, Konstantina Orlandatou and Georg Hajdu

1 Introduction

The following text represents the results of over two decades of research into the Bohlen-Pierce scale, which started when in 1991 one of the authors, Georg Hajdu, realized this non-octave scale on Adrian Freed's Reson8 synthesizer in a computer science seminar at UC Berkeley. It further sharpened his awareness of its significance when Manfred Schroeder, world-famous acoustician and close friend of Mathews' and Pierce's approached him about the scale at the ICMPC conference[1] in Los Angeles in the same year. He stayed true to the scale by using it for a scene in his opera *Der Sprung – Beschreibung einer Oper* written in 1994. An acoustic Bohlen-Pierce clarinet would have been badly needed, but it took another ten years, before someone capable of tackling the project was found: Stephen Fox, physicist and clarinet builder from Toronto finally offered first BP clarinets for sale in early 2006[2]. Two concerts that ensued ushered in the acoustic BP era, one performed by the Ontario-based TranSpectra group[3] on March 20, 2008 including a piece by Todd Harrop and the other performed by clarinetist and co-author Nora-Louise Müller and her friend, the late Anna Bardeli, with pieces by composition faculty of the Hamburg University of Music and Theater (HfMT)[4]. Both concerts were very encouraging and inspired Hajdu to propose to the Goethe Institute in Boston a symposium entirely dedicated to the scale, to which all the key figures in the field were supposed to give an account of their research, developments and artistic work[5]. The proposal was accepted and the symposium held from March 7 – 10, 2010 co-organized by Northeastern University, the Berklee College of Music, the New England Conservatory, the Boston Microtonal Society and the Boston Goethe Institute with the generous help of Anthony De Ritis, Julia Werntz, Richard Boulanger and Annette Klein. It featured three concerts in different locations with over 20 premieres as well as 20 presentations, by David Wessel, Clarence Barlow, Curtis Roads, Richard Boulanger, Psyche Loui, Ron Sword, Paul Erlich, Larry Polanski, Manfred Stahnke, Max Mathews and Heinz Bohlen (the latter two via Skype due to health reasons), and Georg Hajdu, among others. The presentation by Mathews, a close collaborator of the scale's co-discoverer John R. Pierce was supposedly one of his last public appearances; he died a year later. One of the highlights of the conference was the premiere of *Pinball Play* by

1 http://www.icmpc.org (accessed June 01, 2014).
2 http://www.sfoxclarinets.com/BP_sale.html (accessed June 01, 2014).
3 http://www.transpectra.org (accessed June 01, 2014).
4 http://mmm.hfmt-hamburg.de/index.php?id=bohlen-pierce-klarin (accessed June 01, 2014).
5 http://bohlen-pierce-conference.org (accessed June 01, 2014).

Clarence Barlow for four BP clarinets played by Amy Advocat, Ákos Hoffmann, Tilly Kooyman and Nora-Louise Müller standing in the four corners of Northeastern University's beautiful Fenway Center. The symposium had a huge impact on the microtonal communities, its ripples still perceivable today. Subsequently, quite a number of Bohlen-Pierce-related concerts and presentations were organized in various parts of the world which also inspired us to start a course in Bohlen-Pierce theory and ear training at the HfMT—something that had been already envisioned by Mathews, Pierce and others in the 1980's[6].

This slightly sketchy study is in no way meant to replace the information given by Heinz Bohlen himself who is maintaining an extensive website[7]. Section 2 is an account of research on the history and the basics of the scale performed by Nora-Louise Müller. In section 3, Georg Hajdu establishes the foundations of a BP theory, which—albeit in a rather fragmented and incomplete stage—could be the point of departure for a more intricate study. It also takes cognitive and quantitative aspects into consideration such as sensory dissonance curves and Clarence Barlow's harmonicity function. Sections 4 and 5, written by Konstantina Orlandatou with Georg Hajdu cover the ear training lessons and psychological experiments that were carried out at the HfMT with a brief discussion of the results we obtained. Finally, section 6, again by Nora-Louise Müller, deals with the existing and ever growing family of BP instruments.

Conducting artistic and/or theoretical research in the Bohlen-Pierce scale, we find ourselves in a remarkable situation: Inventing in a matter of a few years something that, in case of the chromatic tuning and its notation took a millennium to develop into its current form, is similar to a branch of biology called synthetic biology, which aims at developing novel biological systems from the lessons learned from nature. Likewise, we are attempting to artificially (re)construct a situation analogous to the long development of Western music, the result of centuries of trial and error codified by generations of theorists. Doing the same with the BP scale means going back in time and arbitrarily choosing a historic template from which to model our analogies. In other words, we could build a theory or practice akin to Palestrina or Bach counterpoint, 4-part choral harmonizations, modal mixtures à la Messiaen or 13-tones rows according to Schoenberg or Webern. Even a new Jazz theory could be established or compositions resembling avantgardist music written by Ferneyhough or Lachenmann, let alone compositions in the tradition of the French spectralists. And there is BP microtonality too: Just intonation (see Todd Harrop's chapter in this book) and equal temperament with its notable subdivisions into 3 and 5[8]. One might ask why. So instead of creating a new prescriptive music theory, the examples given should be considered as a list of possibilities which the reader may

6 Max V. Mathews, John R. Pierce, Alyson Reeves and Linda A. Roberts: Theoretical and experimental explorations of the Bohlen–Pierce scale. Journal of the Acoustical Society of America 84 (1988): 1214.

7 http://www.huygens-fokker.org/bpsite/ (accessed June 01, 2014).

8 By sheer coincidence, the equal division of the twelfth (the 3:1 ratio) into 65 steps (or in "microtonese" 65ed3), with a step size of 1902¢/65 = 29.26¢ is practically identical to the equal division of the octave into 41 steps, or 41edo, with a step size of 29.27¢)—a well-known tuning that also features a diatonic pitch set!

use as a starting point for his or her own explorations of the scale.

2 *The Bohlen-Pierce Scale: Basics, History and Notation*

The Basics

The Bohlen-Pierce (BP) scale is based on the perfect twelfth as its harmonic frame, dividing it into 13 steps. Its base triad has the ratios 3:5:7. The frequency ratio of the perfect twelfth is 3:1, and due to this (triple) ratio the frame interval of the BP scale has been named a tritave[9].

The single notes of the scale can be derived by stacking ten 3:5:7 triads on top of one other. (This method is analogous to the derivation of the Western standard system and the circle of fifths.)

The individual proportions in the 3:5:7 chord in relation to the root are:

1:1 – 5:3 – 7:3 – 3:1, with the last proportion being the repetition of the root note a tritave higher (3:1). The first inversion of the 3:5:7 chord is 5:7:9 (the former root note now a tritave higher; 1:3 = 3:9), and the single intervals are 1:1 – 5:7 – 5:9. Accordingly, the second inversion is 7:9:15 or 1:1 – 9:7 – 15:7.

The 3:5:7 chord and its two inversions form seven of the 13 scale tones:

1:1 – 9:7 – 7:5 – 5:3 – 9:5 – 15:7 – 7:3 – (3:1)

The complete just version of the scale contains the following ratios, all derived from the odd numbers 3, 5, 7 and their multiples:

scale step	ratio		scale step	ratio
0	1:1		7	9:5
1	27:25		8	49:25
2	25:21		9	15:7
3	9:7		10	7:3
4	7:5		11	63:25
5	75:49		12	25:9
6	5:3		13	3:1

Table 1: The BP scale consists of intervals made of odd integer ratios

9 Max V. Mathews, John R. Pierce and Linda A. Roberts: Harmony and New Scales. In: *Harmony and Tonality*. Royal Swedish Academy of Music. Stockholm, 59-84, 1987.

In the equal-tempered version, the tritave (1902¢) is divided by 13, yielding a step size of 146.3¢.

The frequencies of steps in this scale, 13 divisions of 3:1, can be calculated by multiplying the frequency of one note by the 13th root of 3 = 1.0882. For example, if the assumed starting note is 440 Hz, the next BP note is $440 \cdot 1.0882$ = 478.8 Hz.

History and Origin of the Bohlen-Pierce Scale

One interesting fact about the BP scale is that it neither developed over hundreds of years unlike other scales and tunings; nor was it discovered by one person at one time. It was rather discovered three times, by three people independently from each other: Heinz Bohlen, Kees van Prooijen and John R. Pierce.

The first person to discover in 1972 what later would be referred to as the Bohlen-Pierce scale was German Heinz Bohlen. Bohlen—no musician, but rather a microwave electronics and communication engineer—asked an unusual question which might have seemed naive to many of the music theorists and composers he was in contact with: Why is the octave the basis of our musical system?

Bohlen wondered about the origin of our traditional system. From what he knew, composers exclusively used a tuning based on the octave, divided into 12 steps. He got curious about the reasons why this system is used and not any other, and what might make it superior to other possibilities. As he didn't get satisfying answers from his professional composer and music theorist friends, he started researching on his own. He finally found an explanation that satisfied him: Combination tones seemed to be the reason why the Western standard system of twelve steps within an octave had been in use for centuries[10]. The major triad (4:5:6) owes its almost unique harmonic properties to combination tones, and the Western standard system developed because it apparently was the only solution that allowed to fit as many instances of it as possible into the framework of an octave.

10 Hermann von Helmholtz, *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*, Braunschweig, 1863.

Fig. 1. Detail of the very first BP instrument: An electronic organ built by Heinz Bohlen

Bohlen had found an answer to his question. This stimulated his curiosity, and he played around with numbers to try out other solutions than 12edo—12 equal division of the octave. At that time, in 1972, his research was purely theoretical. He did not have an instrument to try out triads and scales since most acoustic instruments – e.g. piano or wind instruments – could not be used for systems other than the Western standard system, and synthesizers were out of the financial reach of most private persons. However, Bohlen came up with an idea. Fueled by the concept of cubic difference tones, he replaced the 4:5:6 triad within an octave by a triad with the pitch relations 3:5:7 within the frame of the interval 3:1—the perfect twelfth. Building inversions and filling the gaps nearly equidistantly, he thus derived all steps of his "13-step scale"[11]. Using *continued fractions,* he later confirmed his findings from a mathematical point of view. In 1973, he finally finished building his own home organ for his scale, and for the first time he was able to hear what he had found about 1 ½ years earlier.

Kees van Prooijen is a Dutch software engineer who in the late 1970s discovered the same scale when he was investigating about equal temperaments and continued fractions[12]. He published a paper in 1978 in which his approach to establishing the scale was elaborated. Van Prooijen, however, did not publish explicitly about his

11 Heinz Bohlen: 13 Tonstufen in der Duodezime. *Acustica* 39, no. 2 (1978): 76–86.

12 Kees van Prooijen: A Theory of Equal-Tempered Scales. In: *Interface*, 7 (1978): 45-56.

13ed3 scale[13] and thus was overlooked as the actual second discoverer of the scale.

John R. Pierce was a multitalented American engineer. To be more exact, his profession was the same as Heinz Bohlen's: He was a microwave electronics and communication engineer. In the 1940s he had been part of a group to develop the transistor – and had given the device its name – , and when the first TV satellite was installed in 1962, a big part of this achievement was to Pierce's credit. He furthermore had a special interest in computer music and psychoacoustics which much later in his life – after retiring from his engineering job – led him to work at the well-known Center for Computer Research in Music and Acoustics (CCRMA) at Stanford University.

Pierce found the scale by serendipity. Max Mathews had told him about an intonation perception experiment that he and his colleague Linda Roberts had been leading earlier. They were researching about preferences of listeners regarding equal-tempered vs. pure major or minor third[14]. In their experiment they exposed subjects to the traditional major and minor chord in their equal-tempered and just versions. They added the two non-traditional triads 3:5:7 and 5:7:9. Other than Bohlen, they had electronic synthesizers in their lab and could easily generate these unusual harmonies. When John Pierce heard about the experiment in 1984, he got curious about the new triads. He and Mathews wondered which equal-tempered scales would accommodate this triad and found no matching subdivision of the octave; Pierce's idea was to base the scale on 13 root of 3, and it worked. The factor 3 was inherent in the chord, and the 13th root simply was chosen by trial and error[15]. Only some years later he heard about Heinz Bohlen and his earlier discovery of the same scale.

Just as Bohlen who had used a square wave for his electronic organ, Pierce —keenly aware of the fact that the timbre of a sound plays a significant role in the perception of consonance and dissonance in a scale or chord[16]—argued that a timbre of odd harmonics would be suitable for the "Pierce scale"[17].

Bohlen-Pierce Notation

The practice of BP music brings up questions about notation. The traditional diatonic notation on five lines turns out to be not ideal since it is connected to the Western standard system and other octave related scales in the interpreter's mind; notating a

13 13ed3 means 13 equal divisions (ed) of 3:1; accordingly, the Western standard system in its equal-tempered version can be referred to as 12ed2 since it is 12 equal divisions of 2:1.

14 Max V. Mathews, John R. Pierce and Linda A. Roberts: Harmony and New Scales. In: *Harmony and Tonality*. Royal Swedish Academy of Music. Stockholm, 59-84, 1987.

15 ibid.

16 John Pierce: Attaining Consonance in Arbitrary Scales. *Journal of the Acoustical Society of America*, 40 (1966): 249.

17 Suggested further reading: William A. Sethares: *Tuning, Timbre, Spectrum, Scale*, New York: Springer, 1999. Chapter 6 explores the interaction between harmony and timbre in the BP scale.

non-octave scale within this system easily leads to irritation about step sizes and harmonic frames. It is possible to notate BP pitches by using eighth-tone or sagittal[18] accidentals, but it is unsatisfying because musicians do not easily recognize the structure of the non-octave BP scale when mapped onto a diatonic octave-based notation system.

The question as to how to notate the BP scale is essential due to the fact that current BP instruments were designed under aspects specific to each instrument and thus retaining some of the characteristics of its 12edo sibling (see section 6 on the Bohlen-Pierce clarinets). Different scenarios require different approaches to notation: For a conductor directing an ensemble it may be most useful to read from a score written in standard Western diatonic notation with microtonal accidentals. For a BP clarinetist, a system representing the fingerings he or she is acquainted with would be most efficient, and for a composer or theorist it would be useful to read scores in a chromatic BP notation.

Several suggestions of a BP notation have been made e.g. by Manuel op de Coul which will be discussed shortly in this article. We are also proposing the Müller-Hajdu notation, developed by co-authors Nora-Louise Müller and Georg Hajdu, which, for consistency’s sake, we will use throughout the entire article.

Op de Coul Notation

Being amongst the first attempts at creating a notation system for BP music, Manuel op de Coul's notation is widely accepted by BP musicians.

Based on Bohlen’s own approach, op de Coul suggests two five-line staves with treble and bass clefs marked BP. As a reference mode he chooses Lambda[19], and the note names c d e f g h j a b representing the BP scale steps 0, 2, 3, 4, 6, 7, 9, 10 and 12 or interval structure 2 1 1 2 1 2 1 2 1; all other key areas and modes and are notated by adding sharps or flats.

The staff conveniently covers, on a five-line system, the range of two tritaves so that clefs do not need to be changed very often. Asymmetric keyboard layouts with 9 white and 4 black keys can also be easily learned. However due to the similarities of note names, clefs and staves to standard notation, musicians may get mixed up. For instance, the BP interval c–c is meant to be a tritave but is notated like the traditional c'–e'' which causes a double confusion in terms of the meaning of the interval: c–c is usually considered an octave, but here it indicates a tritave (perfect twelfth), plus one would read a tenth in standard notation which has nothing to do with either. In addition, there is no clear statement by op de Coul about the exact frequency of the bottom c note. Should a tempered c be assumed, A440 would not be part of the scale and thus the notation unusable for the BP clarinets.

18 http://sagittal.org (accessed June 01, 2014).

19 Lambda is a diatonic mode in BP. Diatonic modes are shown in table 4 in section 3.

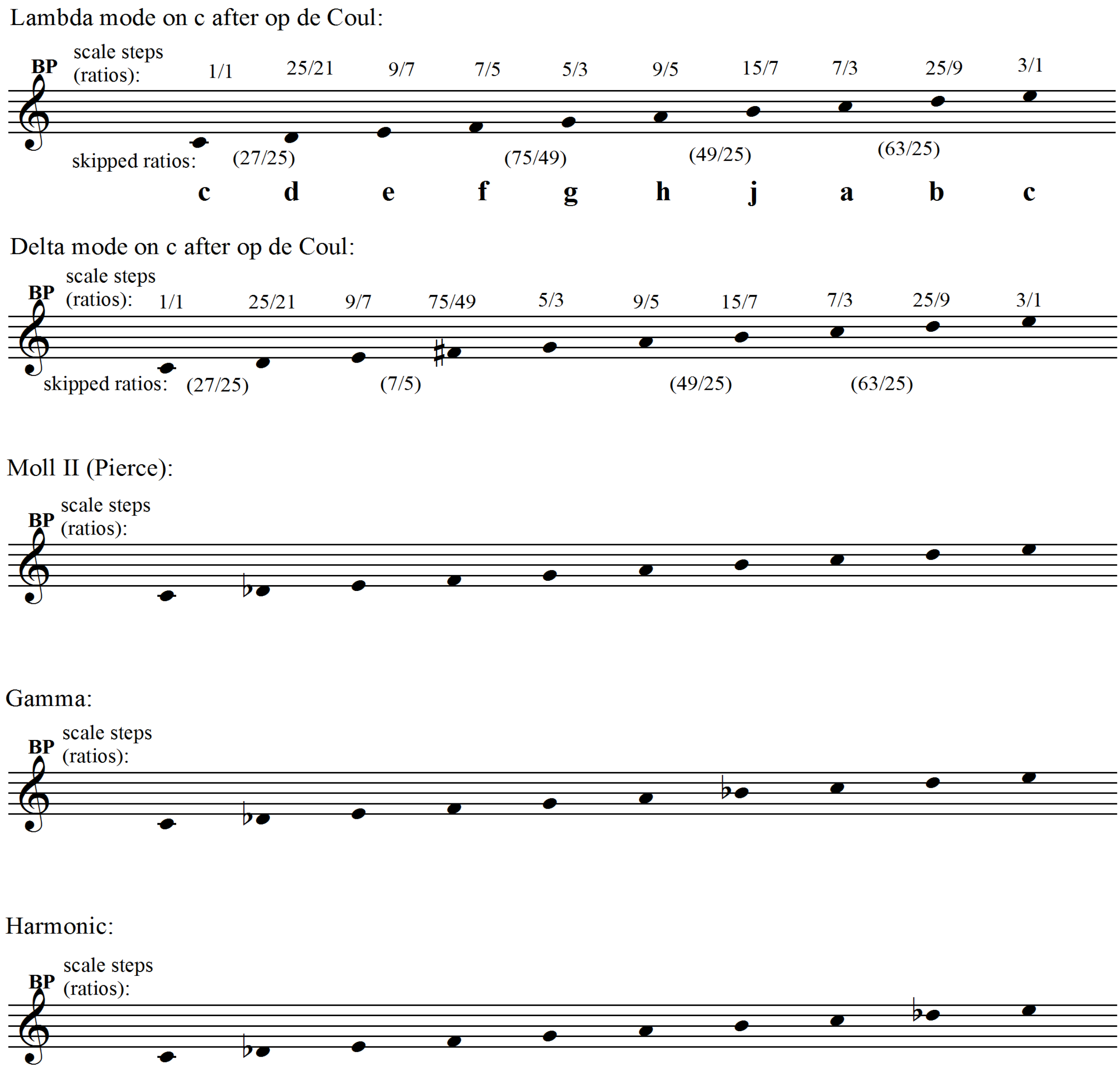

Fig. 2: Several BP modes in op de Coul's five-line notation.

The confusing nomenclature notwithstanding, op de Coul's notation has the great potential of being established specifically as standard notation for keyboard instruments. A notation system using accidentals implies agreement on a reference scale—in this case C Lambda—which can be identified with a certain keyboard layout. As on a traditional keyboard, notes with accidentals would be represented by black keys while C Lambda would be achieved by playing on white keys only. Elaine Walker's work for instance perfectly connects to op de Coul's ideas. Walker has been thinking about BP keyboard layouts since the early 2000s, originally using a layout based on Lambda mode—following op de Coul's suggestion. It turned out to make orientation difficult for the player due to its very regular distribution of black and white keys. Walker thus decided to rather use a Dur II layout—or Gamma layout, which looks the same. This layout, which was originally also realized on Bohlen's organ, provides a clearer arrangement through its slightly irregular distribution of black keys within a tritave; yet it only differs from Lambda layout in one aspect[20].

Looking at it carefully, it becomes obvious that, provided the use of a matching keyboard layout, op de Coul's notation is comfortable and easy to learn for an experienced keyboard player.

Nevertheless the use of a reference mode and accidentals consequently is critical for players of other instruments, because instruments (like string and wind instruments) may be constructed under very different aspects and not based on a reference scale.

The Müller-Hajdu Notation

The Müller-Hajdu notation is a chromatic notation with each of the 13 scale steps on or between lines. It does not use accidentals and hence is independent from a reference mode or specific instrument layout. It thus resembles other chromatic notation systems invented in 20th century such as the Hauer-Steffens notation or Klavarskribo[21].

Note names are taken from the last 13 letters of the alphabet, N-Z, to avoid confusion with traditional note names a-g. The staff consists of six lines. This makes it possible to notate the range of two tritaves, the standard range of the BP clarinet, using four ledger lines above and below staff, which is reasonably practicable for musicians:

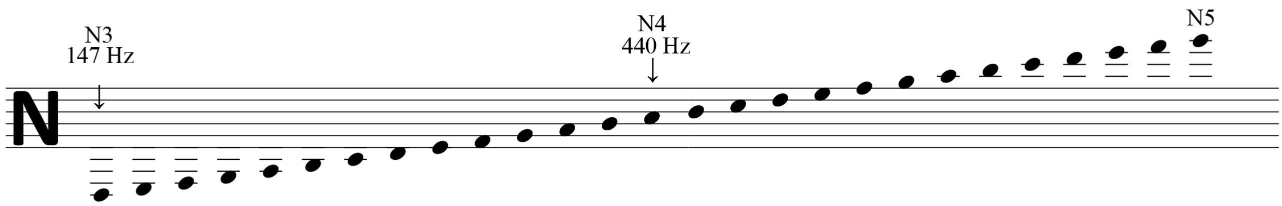

Fig. 3. Bohlen-Pierce chromatic notation:
The range of two tritaves conveniently fits on a 6-line staff in N clef

Most acoustic BP instruments share an important tone with the standard system, the sounding a' (440-442 Hz). This note is located in the middle of the staff. The system can on first sight be distinguished from the traditional five-line staff; there is no danger of mixing up the two notation systems. A system with six lines is still readable by musicians; more staff lines, though, may lead to problems due to increasing visual complexity and thus latency in reliably identifying scale steps.

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20 A detailed description of the two keyboard layouts can be found on Elaine Walker's webpage. She has been building BP keyboard instruments since 2010. http://www.ziaspace.com/_microtonality/BP (accessed June 01, 2014).

21 http://musicnotation.org/systems/ (accessed June 01, 2014).

The N clef perfectly accommodates the range of a Bohlen-Pierce clarinet. For lower instruments or voices the T or the Z clef may be chosen instead:

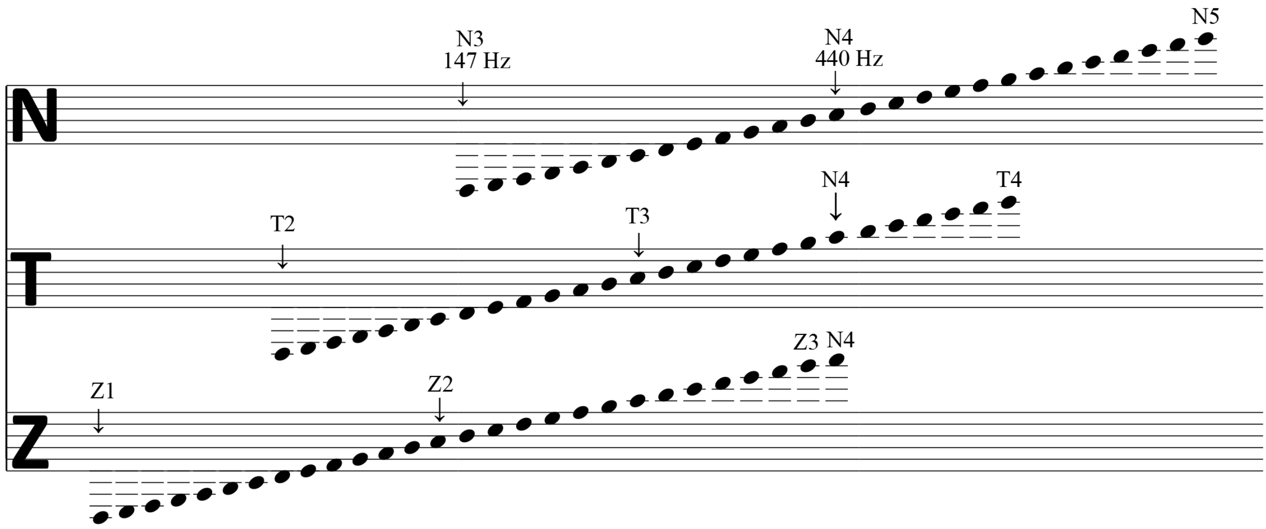

Fig. 4: A wide range of notes can be represented by employing Z, T and N clefs. Note how visual illusions regarding the positions of N can guide the musicians while reading this notation.

Please note that the visual effect between N and Z clefs is similar to that between treble and bass clef in the standard notation system: The two notes on the first ledger lines above and below staff respectively have the same pitch (T3). This system is comfortable for Western musicians through its subtle use of a visual illusion: For example, in N clef, N3 is located on the fourth ledger line below staff; the same pitch in Z clef is situated on the third line from top. The eye is tricked and recognizes these two notes as the same pitch, since on first sight it looks like a *d* in traditional bass or violin clef. Hence the musician intuitively recognizes the tritave relation between the two notes since it visually reminds him of an octave relation in the 12-tone system. This effect works throughout the N and Z clef systems. The visual effect of the T clef in relation to the N and Z clefs is similar.

For the BP tenor clarinet an additional U clef – six BP steps below the N clef – would be suitable. A clarinetist would only have to learn one clef because the relation between N and U clef is the same as between soprano and tenor clarinet (i.e. same fingerings for same note lines and spaces). This comfort is more than useful for the player because the danger of getting mixed up in performances is eliminated.

Stacking just twelfths covering the range of a Bösendorfer Imperial grand piano (with the exception of its very top note), we find 5 tritaves starting on anchor tones C0 (-8¢), G1 (-4¢), d3 (-2¢), a4, e5 (+2¢) and b6 (+4¢), numbered 1 through 5. We can use the T clef as an all-purpose clef to cover all tritaves, specifying its range by adding the tritave number as an index: T1, T2, T3, T4 and T5[22]. By definition, a T clef without index is to be considered a T3 clef. Likewise, we can also add indexes to the

22 Alternatively, T clefs could also be designated with the names of their anchor tones, such as T_C, T_G, T_D, T_A, T_E.

BP note names to provide tritave designations (for example: R2 or Z4).

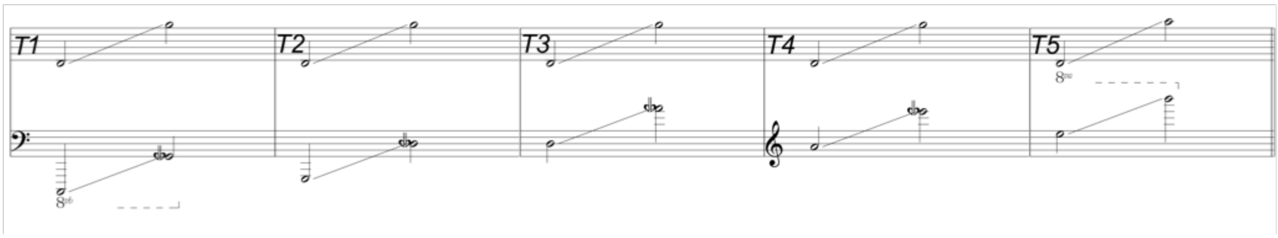

Fig. 5: The T clef can also be used for tritave transpositions. T3 or just T refers to the middle tritave.

It appears that composers, conductors and musicians each have different preferences when it comes to notation. Composers may prefer to learn a simpler BP clef system than the proposed N, T and Z clefs and hence may want to go with the suggestion of the transposing T clefs (see Fig. 5). Musicians need either an instrument-specific notation or BP notation in the clef that works best for their instrument (e.g. N clef for BP clarinet or soprano voice; T clef for cello or tenor voice), or both if they need to communicate with their colleagues in ensemble rehearsals. Conductors, in a third case, are in a particular situation when leading a BP ensemble rehearsal, as they need to reliably identify written as well as sounding pitches and communicate about this with the musicians. Personally, they may prefer a standard microtonal notation (such as eighth-tone notation), but, obviously, communication about pitches is only possible by the use of a common notation system such as the Müller-Hajdu notation. For this reason, all scores and ensemble parts should be available in this notation, as well as in a notation that may be personally preferred by each musician or conductor. In case of a larger ensemble the authors suggest that each musician get an instrumental part containing both Müller- Hajdu notation and an instrument-specific notation, each written on two individual staves. The conductor, if there is one, is in need of both the Müller-Hajdu notation and a standard microtonal notation, which may be most easily realized by the use of an electronic score that allows switching between both notation systems.

The following figure shows BP notation in all notation styles implemented in MaxScore: Eighth-tone and Helmholtz-Ellis JI pitch notation on the first two staves, followed by the 6-line T and N clefs, followed by the fingering notation for the BP soprano and tenor clarinets, and finally MIDI keyboard notation with c4 +22¢ somewhat arbitrarily mapped onto key number 60 (middle-c).

Fig. 6: The BP scale in different views generated by MaxScore[23]

23 http://www.computermusicnotation.com

3 Bohlen-Pierce Music Theory

Acoustics

The just intonation version of the Bohlen-Pierce scale is composed exclusively of intervals with odd number ratios. It is not a far stretch to assume that spectra consisting of odd harmonics only are therefore more suited for Bohlen-Pierce music than spectra with the full set of even and odd harmonics. And indeed, there are a number of spectra that exhibit the desired quality:

- Instruments with stopped pipes such as the clarinet
- Idealized waveforms such as square and triangle waves

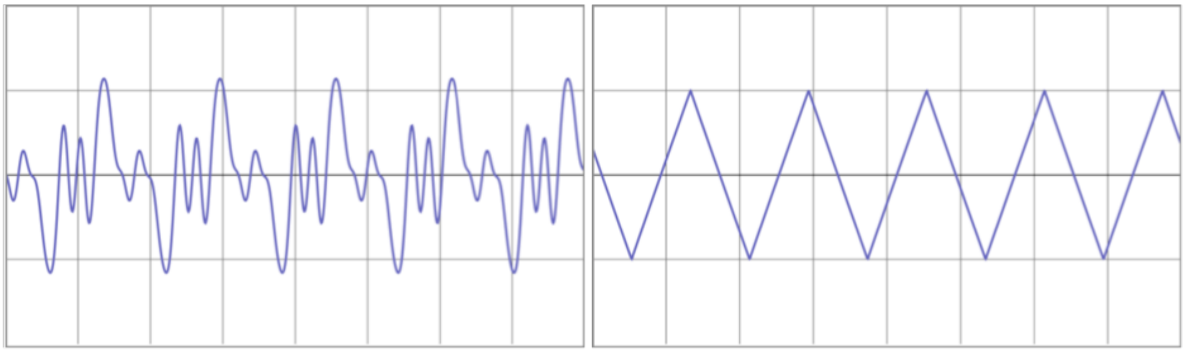

Fig. 7. Left: Clarinet-like spectrum synthesized from 6 components with maximum amplitudes of f0 = f3 = 1., f5 = 0.43, f6 = 0.52, f7 = 0.16 and f8 = 0.23 (note that in the clarinet spectrum even harmonics are always present, albeit to a lesser degree). Right: Triangle wave

A scale or a tuning?

The Bohlen-Pierce scale has occasionally been called a macrotonal scale as its (average) step size of 146.3¢ is almost 50% larger than the chromatic semitone[24]. With 8.2 steps at the octave it more closely resembles the whole-tone scale (6 steps) than the chromatic scale (12 steps). In contrast to microtonal tunings with step sizes smaller than 100¢—often functioning as pitch sets from which to construct the actual scale or mode—the BP scale can be satisfactorily used as a scale without needing to omit steps, while some of the BP 9-tone modes with their 4 BP seconds (at nearly the same size as a tempered minor third) can feel a bit unbalanced, particularly with the members of the Gamma family which are allowed to contain two consecutive BP seconds. Hence, taking this into consideration, it was one of our main motivations to create a chromatic notation system which wouldn't favor certain scale tones over others.

Yet, a good reason for using 9-modes when working in a tonal context lies in their asymmetry, which is a prerequisite for generating a sense of tonal direction and hierarchy (something we have elaborated in chapter 5). We have therefore in our

24 https://xenharmonic.wikispaces.com/Bohlen-Pierce (accessed June 01, 2014).

courses focused on two modes, the Lambda and Pierce modes, the former being considered an equivalent to the major scale or Ionian mode and the latter being characterized by its mirror symmetry, and thus resembling the Dorian mode.

BP intervals

Mathematical theories claim that concordance or harmonic consonance of intervals can be directly derived from their prime factorization. The smaller the exponents in the ratios the more concordant the impression they create. To illustrate this fact, we have color-coded the 13 intervals by using the three primary colors red, green and blue (rgb) representing prime factors 7, 5 and 3. To make the results more convincing, the rgb values have somewhat arbitrarily been set to 255 for a factor raised to the power of 0 (thus 1), to 205 for a factor raised to the power of 1, to 45 for a squared factor, and to 0 for a cubed factor. The emotive qualities of the intervals have also been judged by three participants of the 2013/14 BP course which appear on the right of the table:

0	N	0	0	1:1	$3^{0} \cdot 5^{0} \cdot 7^{0}$		neutral	lonely	settled
1	O	146.3	133.2	27:25	$3^{3} \cdot 5^{-2} \cdot 7^{0}$		dirty	uptight	tense
2	P	293.6	301.8	25:21	$3^{-1} \cdot 5^{2} \cdot 7^{-1}$		sad	depressed	soft
3	Q	438.9	435.1	9:7	$3^{2} \cdot 5^{0} \cdot 7^{-1}$		open warning	wide open	hard
4	R	585.2	582.5	7:5	$3^{0} \cdot 5^{-1} \cdot 7^{1}$		neutral	confused	round
5	S	731.5	736.9	75:49	$3^{1} \cdot 5^{2} \cdot 7^{-2}$		empty	rigid	vibrant/lost
6	T	877.8	884.4	5:3	$3^{-1} \cdot 5^{1} \cdot 7^{0}$		lush	light&soft	calm
7	U	1024.2	1017.6	9:5	$3^{2} \cdot 5^{-1} \cdot 7^{0}$		disturbed	gushing	warm
8	V	1170.5	1165.0	49:25	$3^{0} \cdot 5^{-2} \cdot 7^{2}$		empty	mellifluous	vibrant/solid
9	W	1316.8	1319.4	15:7	$3^{1} \cdot 5^{1} \cdot 7^{-1}$		like tritone	tight	ambitious
10	X	1463.1	1466.9	7:3	$3^{-1} \cdot 5^{0} \cdot 7^{1}$		friendly	present	smooth
11	Y	1609.4	1600.1	63:25	$3^{2} \cdot 5^{-2} \cdot 7^{1}$		gentle caress	radiant	scratchy
12	Z	1755.7	1768.7	25:9	$3^{-2} \cdot 5^{2} \cdot 7^{0}$		tension	bold	into the firing line
13	N	1902.0	1902.0	3:1	$3^{1} \cdot 5^{0} \cdot 7^{0}$		bright victory	job done	empty

Table 2. The intervals of the BP scale according to step number, note names, size of the tempered and just intervals, interval ratio, prime factorization, color coding and individual verbal descriptions.

Due to the values chosen discordant intervals appear darker than concordant ones, the wolf intervals BP 5th and BP 8th have the darkest hues. Notice that the color scheme also reveals the symmetry axis inverting the intervals at the tritave. It is located between BP 6th and BP 7th.

Harmonic consonance

While sensory consonance (see below) is a passable predictor for the pleasantness of simultaneous musical events[25], it fails at capturing more abstract, mental relationships. Clarence Barlow has therefore developed a formula[26] similar to Euler's Gradus Suavitatis[27] or James Tenney's Harmonic Distance[28] function to quantify harmonic consonance, and whose values are given in Table 3 for the intervals of the BP scale[29].

0	1:1	$3^{0} \cdot 5^{0} \cdot 7^{0}$	∞
1	27:25	$3^{3} \cdot 5^{-2} \cdot 7^{0}$	-0.048
2	25:21	$3^{-1} \cdot 5^{2} \cdot 7^{-1}$	-0.039
3	9:7	$3^{2} \cdot 5^{0} \cdot 7^{-1}$	-0.064
4	7:5	$3^{0} \cdot 5^{-1} \cdot 7^{1}$	0.060
5	75:49	$3^{1} \cdot 5^{2} \cdot 7^{-2}$	-0.028
6	5:3	$3^{-1} \cdot 5^{1} \cdot 7^{0}$	0.110
7	9:5	$3^{2} \cdot 5^{-1} \cdot 7^{0}$	-0.085
8	49:25	$3^{0} \cdot 5^{-2} \cdot 7^{2}$	0.030
9	15:7	$3^{1} \cdot 5^{1} \cdot 7^{-1}$	-0.052
10	7:3	$3^{-1} \cdot 5^{0} \cdot 7^{1}$	0.077
11	63:25	$3^{2} \cdot 5^{-2} \cdot 7^{1}$	0.035
12	25:9	$3^{-2} \cdot 5^{2} \cdot 7^{0}$	0.055
13	3:1	$3^{1} \cdot 5^{0} \cdot 7^{0}$	0.375

Table 3: According to Clarence Barlow and other theorists, harmonic consonance is an emerging property of prime factorization and passable predictor of preference ratings. The values appearing

25 One may wonder how well it even predicts the perceived consonance of simultaneous events, as gestalt principles are capable of overriding sensory dissonance sensations, such as in the case of a tempered minor sixth which, to the "naked" ear, is almost always heard as the frame of a first inversion major triad.

26 Clarence Barlow: Two Essays on Theory. *Computer Music Journal* 11, no. 1 (1987): 44-60.

27 Leonhard Euler. Tentamen novae theoriae musicae ex certissimis harmoniae principiis dilucide expositae. St. Petersburg, 1739.

28 John Chalmers: *The Divisions of the Tetrachord*. Frog Peak Music, 1993.

29 Barlow takes all possible interval combination of prime numbers into consideration. Removing prime 2 from this set of numbers, as we have done, may be an interesting intellectual exercise, but it is debatable to what extent we will actually be able to unlearn familiar, in an odd-partials-only-context, intervals that are in the vicinity of BP scale steps, such as the major tenth.

in the right-most column are, to a large extent, in line with the ratings given in chapter 5, the main difference being the BP 2nd and BP 11th, which—despite their high numerical complexity/low harmonic consonance—are perceived, by many a listener in the isolated context of the experiment, as the familiar minor third and major tenth. The negative values, by the way, refer to something Barlow calls polarity, i.e. the tendency of the lower note (positive sign) or upper note (negative sign) to be the root of a dyad .

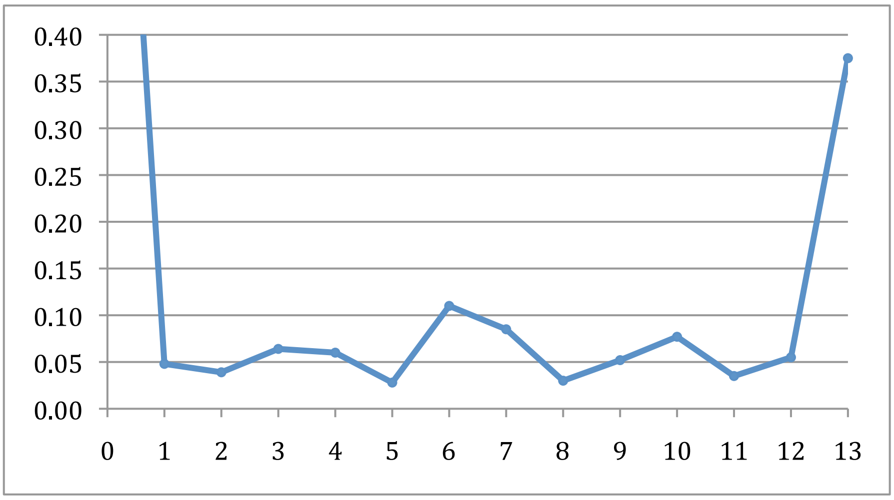

Fig. 8: Absolute harmonic consonance values for the steps of the Bohlen-Pierce scale.

Dissonance curves

Sensory dissonance is a phenomenon associated with the perception of roughness between the partials of one or several tones[30]. The effect is related to critical bandwidth, a notion referring to functional units of inner hair cells involved in the perception of pitch and amplitude. A critical band is said to be approximately 1/3 of an octave in size in the mid to upper frequency range, yet well over an octave in the low frequency range. In the 1960's, Plomp & Levelt[31] as well as Kameoka & Kuriyagawa[32] have performed measurements assessing the dissonance perception of pairs of pure tones and have extrapolated these results for complex, musical tones by summing up roughness values obtained for individual pairs of partials. While more sophisticated approaches to the calculation of sensory consonance exist, we have used the straightforward formula given by William Sethares, taking the first 6 partials and their respective amplitudes into consideration[33]. Using the tempered version of the BP scale, we have calculated dissonance curves for spectra with odd and even partials as well as with odd partials only, their amplitudes rolling off at 88% per partial. As expected, we generally see high sensory dissonance values for BP intervals with even/odd spectra, which can be mainly

30 http://en.wikipedia.org/wiki/Consonance_and_dissonance (accessed June 01, 2014).

31 R. Plomp and W.J.M. Levelt: Tonal consonance and critical bandwidth. *Journal of the Acoustical Society of America* 38 (1965): 548-560.

32 A. Kameoka and M. Kuriyagawa: Consonance theory, part I: Consonance of dyads. *Journal of the Acoustical Society of America* 45, no. 6 (1969): 1451-1459.

33 William A. Sethares: Tuning, Timbre, Spectrum, Scale, p.74.

attributed to the roughness produced by the wolf tones (BP 5th and BP 8th; 75:49 and 49:25) which both are about 30¢ apart from the just fifth and octave (3:2 and 2:1). In the top diagram, the red dots representing the steps of the BP scale, are mostly located on or near a peak. Using odd partials only, is a real game changer though. All of the sudden the red dots are near or in a trough (see diagram on the bottom).

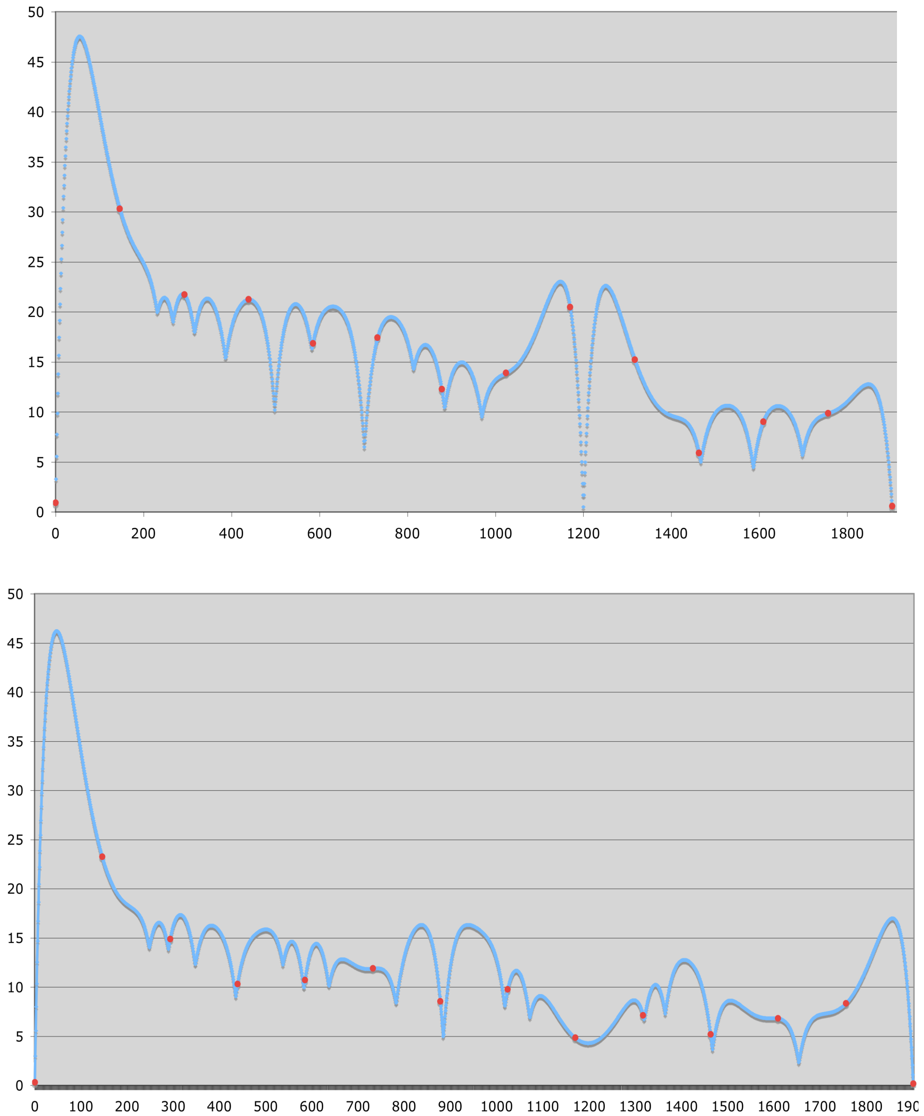

Fig. 9: Sensory dissonance curves for harmonic spectra obtained with William Sethares' algorithm. Top: Even and odd partials. Bottom: Just odd partials.

BP modes and pentachords

Heinz Bohlen has focused on two families (modal cycles) of 9-tone modes, which constitute an analogy to the 7 out of 12 diatonic scales in 12edo. The first modal cycle, also called the Lambda family of modes, is made of 5 small (1) and 4 large (2) steps and emerges when two large steps are prohibited from directly following each other; e.g. 2112 1 2121 for the Lambda scale itself[34].

34 Bohlen calls the other modal cycle the Gamma family of modes. It features two adjacent large

Mode	N_0	O	P	Q	R	S	T	U	V	W	X	Y	Z	N_1
Lambda	X		X	X	X		X	X		X	X		X	X
Walker A	X	X	X		X	X		X	X		X	X		X
Pierce	X	X		X	X		X	X		X	X		X	X
Walker I	X		X	X		X	X		X	X		X	X	X
Harmonic	X	X		X	X		X	X		X	X	X		X
Walker II	X		X	X		X	X		X	X	X		X	X
Dur	X	X		X	X		X	X	X		X	X		X
Delta	X		X	X		X	X	X		X	X		X	X
Walker B	X	X		X	X	X		X	X		X	X		X
Gamma	X	X		X	X		X	X	X		X		X	X
X1	X		X	X		X	X	X		X		X	X	X
X2	X	X		X	X	X		X		X	X	X		X
X3	X		X	X	X		X		X	X	X		X	X
X4	X	X	X		X		X	X	X		X	X		X
X5	X	X		X		X	X	X		X	X		X	X
X6	X		X		X	X	X		X	X		X	X	X
Dur II	X		X	X	X		X	X		X	X	X		X
X7	X	X	X		X	X		X	X	X		X		X

Mode	C	D_b	D	E_b	E	F	G_b	G	A_b	A	B_b	B	C
Ionian	X		X		X	X		X		X		X	X

Table 4: The members of the two families of BP modes (modal cycles) in comparison to the Ionian mode (its color coding being derived from 5-limit prime factorization [primes 2, 3, 5])

Lambda therefore features two non-identical pentachords made of the sequences 2112 and 2121 and set apart by a small step. The only modes with identical pentachords are Dur I (1212 1 1212) and Delta (2121 1 2121).

In contrast to 12edo where the circle of fifths can be derived from a chain of identical tetrachords, chaining identical pentachords with a replication interval of a BP 7th does not generate the circle of BP 10th (see below).

steps like 1212 1 1221 for Gamma. The labels X1 to X7 given to some of its members betrays that they may have not been extensively used for musical purposes (yet).

BP triad and its inversions

The BP triad in its root position (3:5:7), also referred to as the *wide* triad, is made of the intervals BP 0th, BP 6th and BP 10th (thus called a 10-6-chord). Inverting the triad at the tritave (5:7:9) yields the intervals BP 0th, BP 4th and BP 7th (called a 7-4-chord and referred to as the *narrow* triad), while the more harsh sounding second inversion (7:9:15) is composed of the intervals BP 0th, BP 3rd and BP 9th (called a 9-3-chord).

Starting on N one thus gets N, T and X for the root positions triad, N, R and U for the first inversion triad and N, Q and W for the second inversion triad.

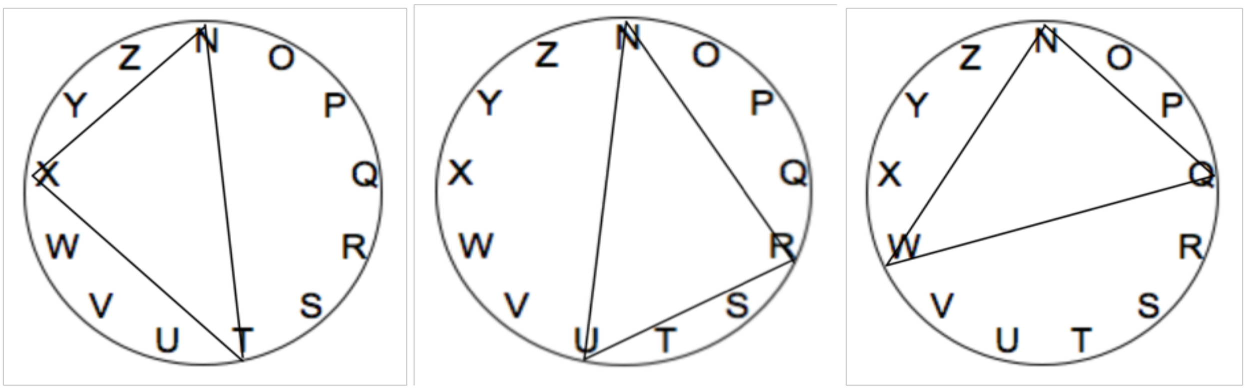

Fig. 10: Clock diagram digram of the BP base triad and its inversions.

Doublings and ranges

Generally, any note of the base triad can be doubled, particularly in root position. In case of an inverted chord, tritave range plays an important role as the resulting virtual fundamental pitch[35] shouldn't be below a certain threshold. Otherwise the chord will quickly sound muddled and dissonant.

35 Ernst Terhardt: Zur Tonhöhenwahrnehmung von Klängen. I. Psychoakustische Grundlagen. *Acustica* 26 (1972) 173-186.

BP circle of 10ths

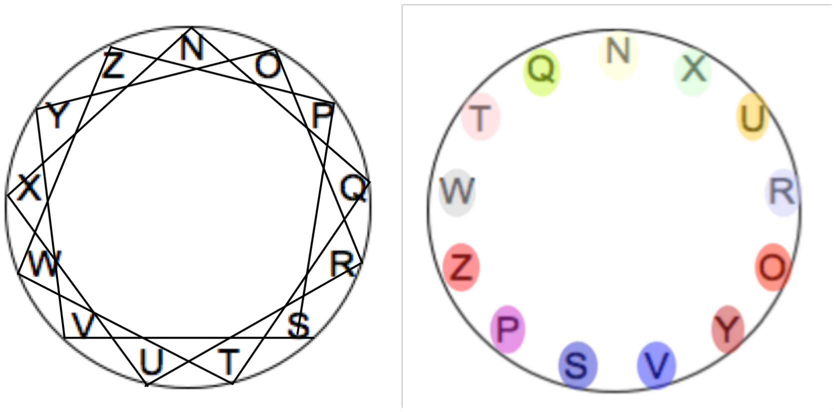

Fig. 11: The circle of 10ths (right) is derived by moving 10 ticks on the 13-tick chromatic dial in clockwise direction, which is the same as going 3 ticks in counterclockwise direction (left).

Stacking BP 10ths (the interval between bottom and top notes of the wide triad), as the basis for a closed circle, we can find remarkable analogies to our traditional 12-tone system:

A subset of 9 adjacent tones will yield the Lambda family of modes with Lambda spanning the notes P to R and Pierce spanning the notes Z to O, both in clockwise motion, all scales starting on N (see Fig. 12).

1. Adjacent key areas share maximum numbers of tones, i.e. X Lambda has more tones in common with N Lambda than with V Lambda.
2. Harmonically complex intervals are located on the opposite side of the fundamental N. The circle on the right shows increasingly dark colors in clockwise and counterclockwise motion with the wolf tones S and V on the bottom; and lighter colors representing less complex intervals on top.

In order to test whether circles based on other intervals than BP 10th (which is just a circle of BP 3rds in counterclockwise motion) would yield similar constellations[36], we constructed closed circles made of BP 7th (BP 6th) and BP 9th (BP 4th). Neither circle revealed any deeper logic in the arrangement of their tones.

36 In the BP scale, all intervals except BP 0th and BP 13th will yield closed circles.

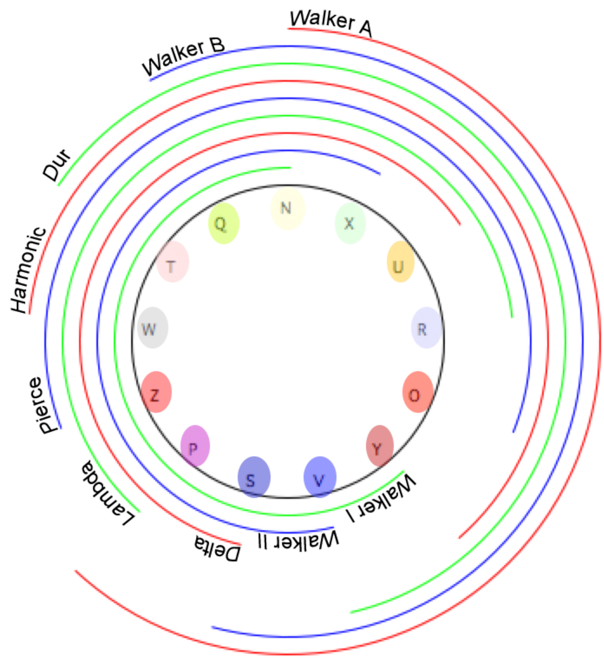

Fig. 12: The Lambda family of modes can be derived by drawing 9-tone segments around the circle of BP 10ths and sorting the notes so that they will all start on N.

BP counterpoint

During the winter semester 2013/14 BP class we undertook the effort to establish a few fundamental rules for 2-part BP counterpoint. Inspired by Dustin Schallert's paper *Tonality, Harmonic Progressions, and Voice Leading with the Bohlen-Pierce Scale*[37] on the Lambda/Pierce hybrid mode (featuring a variable second step degree), we divided its interval set into four categories:

- Perfect consonances: BP 0th, BP 13th , BP 6th and to some extent BP 7th (for which parallel motion should be avoided)[38].
- Imperfect consonances: BP 3rd, BP 4th, BP 7th, BP 9th, BP 10th
- Dissonances: BP 1st, BP 2nd, BP 11th and BP 12th
- Wolf intervals: BP 5th and BP 8th

In comparison to major/minor, we are observing a few peculiarities, which deserve our attention:

1. The consonances BP 7th and BP 9th are close to intervals that are considered dissonant in traditional music theory: m7 and m9.
2. The dissonances BP 2nd and BP 11th as well as BP 5th and BP 8th are close to intervals that are considered imperfect and perfect consonances.

37 http://tinyurl.com/ogd7q4o (accessed June 01, 2014).

38 We are reminded of the writings by James Tenney and others that the notion of a perfect consonance is a historical one and greatly depends on context. Here, with the BP scale, the context is given by the unison, the tritave, the purity of the 5:3 and a number of intervals with low harmonic consonance.

	N_0	***O***	***P***	**Q**	**R**	S	**T**	**U**	V	**W**	**X**	Y	**Z**	**N_1**
N_0	**0**	1	2	3	4	5	6	7	8	9	10	11	12	13
O	1	0	1	2	3	4	5	6	7	8	9	10	11	12
P	2	1	0	1	2	3	4	5	6	7	8	9	10	11
Q	3	2	1	0	1	2	3	4	5	6	7	8	9	10
R	4	3	2	1	0	1	2	3	4	5	6	7	8	9
S	5	4	3	2	1	0	1	2	3	4	5	6	7	8
T	6	5	4	3	2	1	0	1	2	3	4	5	6	7
U	7	6	5	4	3	2	1	0	1	2	3	4	5	6
V	8	7	6	5	4	3	2	1	0	1	2	3	4	5
W	9	8	7	6	5	4	3	2	1	0	1	2	3	4
X	10	9	8	7	6	5	4	3	2	1	0	1	2	3
Y	11	10	9	8	7	6	5	4	3	2	1	0	1	2
Z	12	11	10	9	8	7	6	5	4	3	2	1	0	1
N_1	13	12	11	10	9	8	7	6	5	4	3	2	1	0

Table 5: Interval set of the BP scale. Scale degrees of the Lambda/Pierce hybrid scale are printed in bold. The variants of the second step of the mode are printed in italic.

From these findings we derived the following rules:

1. Perfect consonances as well as the BP 10th and BP 6th may occur on the beginning and the ending note of phrases as well as on weak beats. BP 10th functions much like a third in 12edo and should therefore be used sparsely.
2. The other consonances may occur on strong and weak beats and may be used for parallel motion.
3. Dissonances may occur on weak beats as passing and neighboring notes or on strong beats as appoggiaturas.
4. The rules governing leaps and steps in Renaissance 2-part counterpoint are also applicable: Reversal of direction after a large leap; stepwise motion when the other voice has a leap.

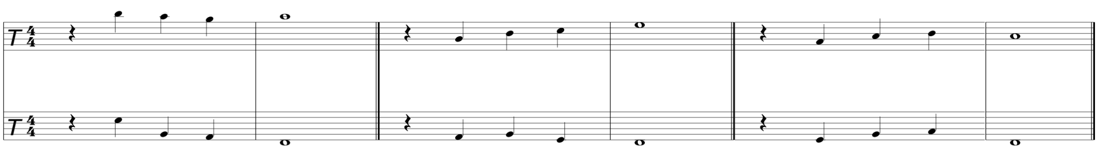

Fig. 13: Possible endings for two-part counterpoint:

On the tritave (left), on the BP 10th, and on the BP 6th

BP functions

Building wide triads on the scale degrees of the Lambda mode will yield 6 regular and 3 irregular dyads (whose interval structure requires adjustment to the mode's interval set). In analogy to major/minor, all regular triads bear functions which, as we will see later, can be used for cadences and sequential progressions.

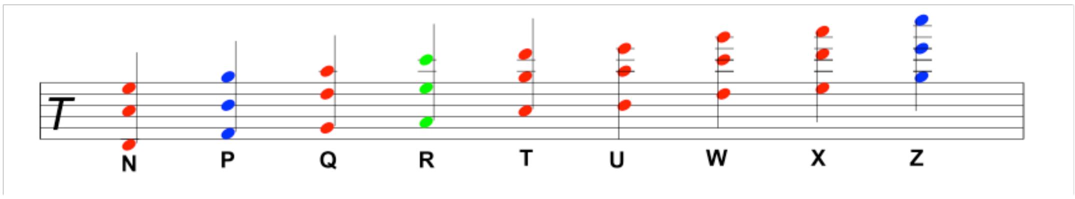

Fig. 14: Triads in the Lambda mode.

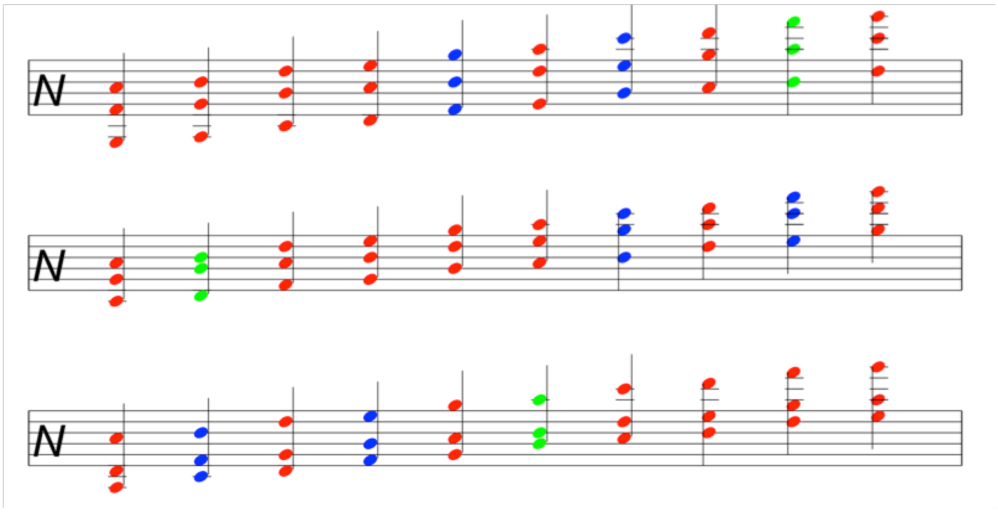

Fig. 15: Harmonization of the ascending Pierce mode by the base triad (top) and its two inversions (middle and bottom). Regular chords are drawn in red, irregular chords are drawn in blue and green.

The example above shows three different three-part harmonizations of the Pierce mode (root position, top; 1st inversions, middle; 2nd inversion bottom) yielding three types of different sonorities:

1. Consonant triads in red
2. Semi-dissonant triads (green)
3. Dissonant triads (blue)

Listening to the results, the unprejudiced listener will observe two things:

1. The roughness of the dissonant chords calls for substitution.
2. Two successive root position triads separated by a large step do not offer enough contrast as they sound very much like members of a

diminished seventh chord.

A pleasing solution only involving “red” chords can be compiled by combining the three harmonizations:

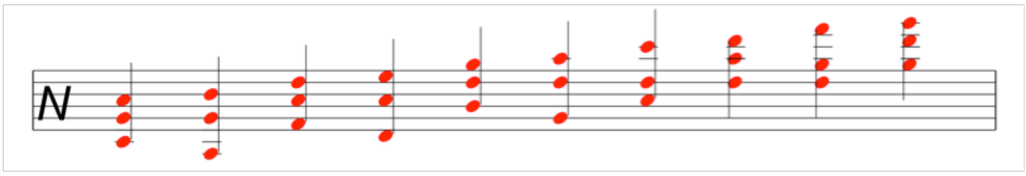

Fig. 16: Harmonization of the ascending Pierce mode by a mix of root-position and inverted triads.

Cadences

Previously, the BP 4th circle was established as an analogy to the traditional circle of fifths. Like in the case of the latter, three adjacent chords Q, N, X form the basis for a subdominant-dominant-tonic-like cadence[39]. Remarkably, all three chords have common notes.

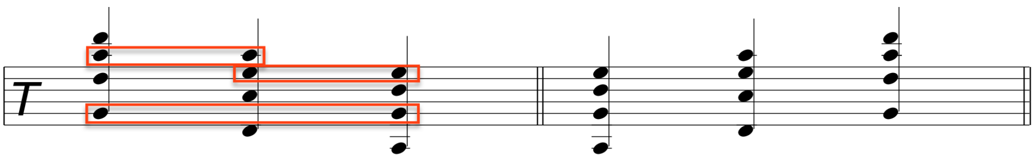

Fig. 17: The N triad and its closely related X and Q triads are the basis for a cadential progression. All three chords have tones in common.

It remained to be seen which of the two possible combinations (Q, X, N or X, Q, N) would yield the strongest cadential effect. After some experimentation with inversions, we found a Q, X^{7}_{4}, N progression that works as a strong authentic cadence. Using X in first inversion has an additional advantage: Its outer voices both move in contrary motion by BP 3rds to the final tonic. In contrast, the other version (X, Q, N) rather sounds like a plagal cadence, the last two chords sharing the scale‘s fundamental. A major difference to traditional harmony is found in the fact that neither Q (= subdominant) nor X (= dominant) contains notes that function as leading tones. Adding them “artificially” creates dissonant sonorities, suggesting that they should be preferably treated as passing or neighboring tones. We may be reminded of the role that the minor seventh played in Western music history before it became an accepted member of dominant harmony. In the Lambda mode we find a U-tonal triad

39 German music theory occasionally also uses the terms *antepenultima*, *penultima* and *ultima* to denote cadential progressions.

that features the second and the second to the last tones of the mode. Its meek diminished-triad character does not commend itself as a strong dominant, though.

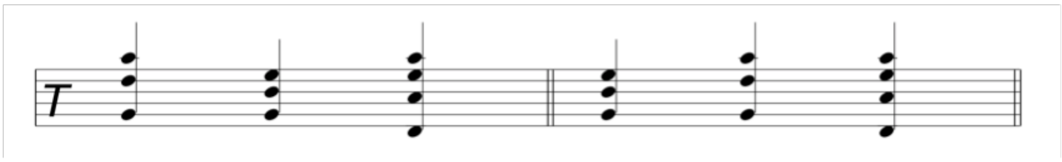

Fig. 18: The N, X and Q triads form authentic and plagal cadences.

Sequences of descending BP thirds

In the Lambda family of modes, a sequence of descending BP 3rds can have up to 6 triads of the same kind. These sequences function very much like descending-fifths sequences in traditional music theory.

Fig. 19: Sequence of 6 descending wide triads.

O and U tonality

The terms O and U tonality were coined by Harry Partch and refer to whether intervals or chords are derived from the harmonic (or overtone) series or its mirror-inverted sibling, the subharmonic (or undertone) series in which the intervals become narrower as one goes down the sequence of tones. Mathews and Pierce also use the terms major and minor to distinguish between these two worlds[40].

Building the U-tonal BP triad (3/3:3/5:3/7) yields a chord, which sounds similar to a traditional diminished triad. It can also be found as 15:21:35 chord in the harmonic series. Likewise the U-tonal 1st inversion chord (7/7:7/9:7/15) which is composed of the ratios 21:35:45 in the harmonic series sounds less concordant due to its BP 9th frame interval, whereas the U-tonal second inversion chord (5/5:5/7:5/9) sounds similar to a dominant seventh chord with its fifth omitted. It can also be constructed from the ratios 35:45:63 contained in the harmonic series. Its tense character can also

40 Max V. Mathews, John R. Pierce, Alyson Reeves and Linda A. Roberts: Theoretical and experimental explorations of the Bohlen–Pierce scale. *Journal of the Acoustical Society of America* 84 (1988): 1214.

be exploited to create dominant-like resolutions into a tonic triad. It would probably make a great secondary dominant.

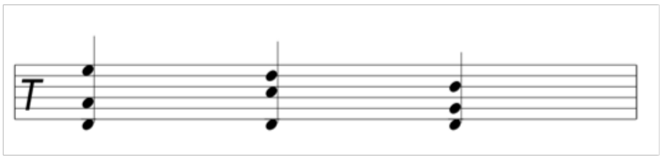

Fig. 20: U-tonal harmonies are derived by mirroring the BP triad and its inversions around the root and transposing them so that they all start with N.

Other harmonies

A major triad 1:3:5 can be constructed from the BP scale when notes beyond a tritave are considered. Composer and BP theorist Elaine Walker uses major triads in her piece *Love Song* which, incidentally, is teeming with allusions to triadic pop harmonies[41].

Fig. 21: The BP scale allows the construction of a traditional major triad.

Chords containing 4, 5 or more different pitches can be built by adding additional notes to the BP triads or using a different approach such as stacking BP 3rds or any other interval, for that matter. For instance, a 6-note chord containing the BP intervals 4, 3, 4, 3, 3 can be derived from Scriabin's Mystic Chord. Note that Scriabin's major thirds and just fourths have been replaced by just one type of interval: the BP 3rd. Nonetheless, built on the 5:7:9 sonority, the chord retains much of the resonant quality, the 12edo version of the chord is known for.

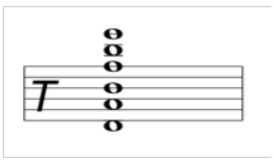

Fig. 22: The Mystic Chord in its BP variant retains much of the resonant quality of its 12edo sibling.

41 http://www.ziaspace.com (accessed June 01, 2014).

Summary and comparison to 12edo

We have taken a closer look at the BP scale's interval content and have derived rules for counterpoint and harmony. In our study it became apparent that the scale, which Curtis Roads calls both more sweet and more sour[42], shows many analogies to 12edo, but also greatly differs from it in many ways.

A prime number set of scale degrees and intervals which due to its indivisibility can't lend itself to the construction of symmetric subsets such as with Messiaen's modes of limited transposition. In Lambda the two pentachords are asymmetrical and can't be used to derive the circle of BP 10th (which we have defined as the analogy to the circle of fifths). In 12edo, the dominant elegantly engulfs the tonic by two neighboring tones, which is unattainable with the chord we identified as dominant in BP, and the scale's wolf intervals need to be treated with special care when building (even advanced) harmonies or counterpoints.

Yet, there are striking similarities too! BP intervals being inverted around the tritave, the triad retains its identity even after inversion. As it will by seen later, the 9-tone modes (particularly Lambda and Pierce) clearly show a hierarchical tonality profile, and O- and U-tonal chords can be used to construct functional harmonic progressions. While BP may never reach the same versatility as 12edo and therefore remain a niche in the musical universe, it still has the potential to develop its own rich musical ecosystem, and we have just begun to understand the rich relationships that we can construct from its 13-tone pitch set, let alone the vast possibilities that microtonal subdivisions may offer to us.

BP spectra

One of the most intriguing topics of contemporary music research is the congruity between spectral and tonal dimensions. As William Sethares pointed out in his book *Tuning, Timbre, Spectrum, Scale* a scale whose interval set was stretched by a certain factor (e.g. using a pseudo-octave of, say, 2.1 instead of 2.0) will sound more compelling when its spectrum is composed of partials shifted by the same amount. We have applied a formula by Mathews and Pierce[43] to the BP scale, which could also be considered a 13edo scale stretched by a factor of 3[44], as early as 1991 using the Reson8 subtractive synthesizer developed by Adrian Freed at IRCAM and CNMAT:

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42 http://bohlen-pierce-conference.org/wp-content/uploads/2010/04/Composing-Purity_14Apr10.pdf (accessed June 01, 2014).

43 Max V. Mathews and John R. Pierce: Harmony and nonharmonic partials. *Journal of the Acoustical Society of America* 68 (1980): 1252.

44 In laymen's terms, the second partial of the stretched spectrum now lies where the 3rd partial of a regular, harmonic, spectrum is.

$$f_n = f_0\, S^{\log(n)/\log(2)}$$

with f_n = frequency of partial, f_0 = fundamental frequency, S = stretch factor, n = partial number

So for instance, the frequency of the 4th partial of a stretched BP spectrum with a fundamental pitch of N3 (d below middle c) would be $f_4 = 146.7\text{ Hz} \cdot 3^{\log(4)/\log(2)} = 146.7\text{ Hz} \cdot 3^2 = 1320\text{ Hz}$, the same pitch as the 9th partial in a regular harmonic series.

In 1994, Hajdu orchestrated a stretched spectrum as the final sonority in the 1st scene of the 1st act of his opera *Der Sprung – Beschreibung einer Oper*, a scene entirely written in the Bohlen-Pierce scale and adjusted the notes to the pitches of the BP scale. In 2007, he further realized the idea by consistently using a stretched spectrum in a piece for two BP clarinets and synthesizer called *Beyond the Horizon*. Due to their inharmonicity and partial spacing the sounds have a tubular bell-like quality. Looking at the dissonance curve obtained from two simultaneous tones, it became obvious that dyads and more complex harmonies played in the BP scale would rather sound tense due to high sensory dissonance. In the top diagram the red dotes representing the pitches of the BP scale are nearly all located on or close to peaks.

Adjustments of the partials to the nearest BP pitches had hardly an effect on the sound of a single tone itself, but audibly effected dyads and more complex harmonies. This was anticipated by the sensory dissonance curve obtained for the adjusted stretched spectra where nearly all notes are located in troughs. While the effect was quite convincing it was still not as dramatic as the two diagrams would suggest. And indeed in one of the concert performances of the piece, the non-adjusted sounds were accidentally used, which had relatively little impact on the overall impression of the piece[45].

45 We hence postulate that sensory dissonance (caused by roughness between partials) may be overrated as a phenomenon governing the sensation of intervals, and that gestalt principles might exert a stronger force, but are more complicated to formalize. The hitherto poorly understood notion of qualia (“redness of red”) seems an appropriate term to describe the unique qualities of intervals regardless of the roughness that their partials may or may not produce.

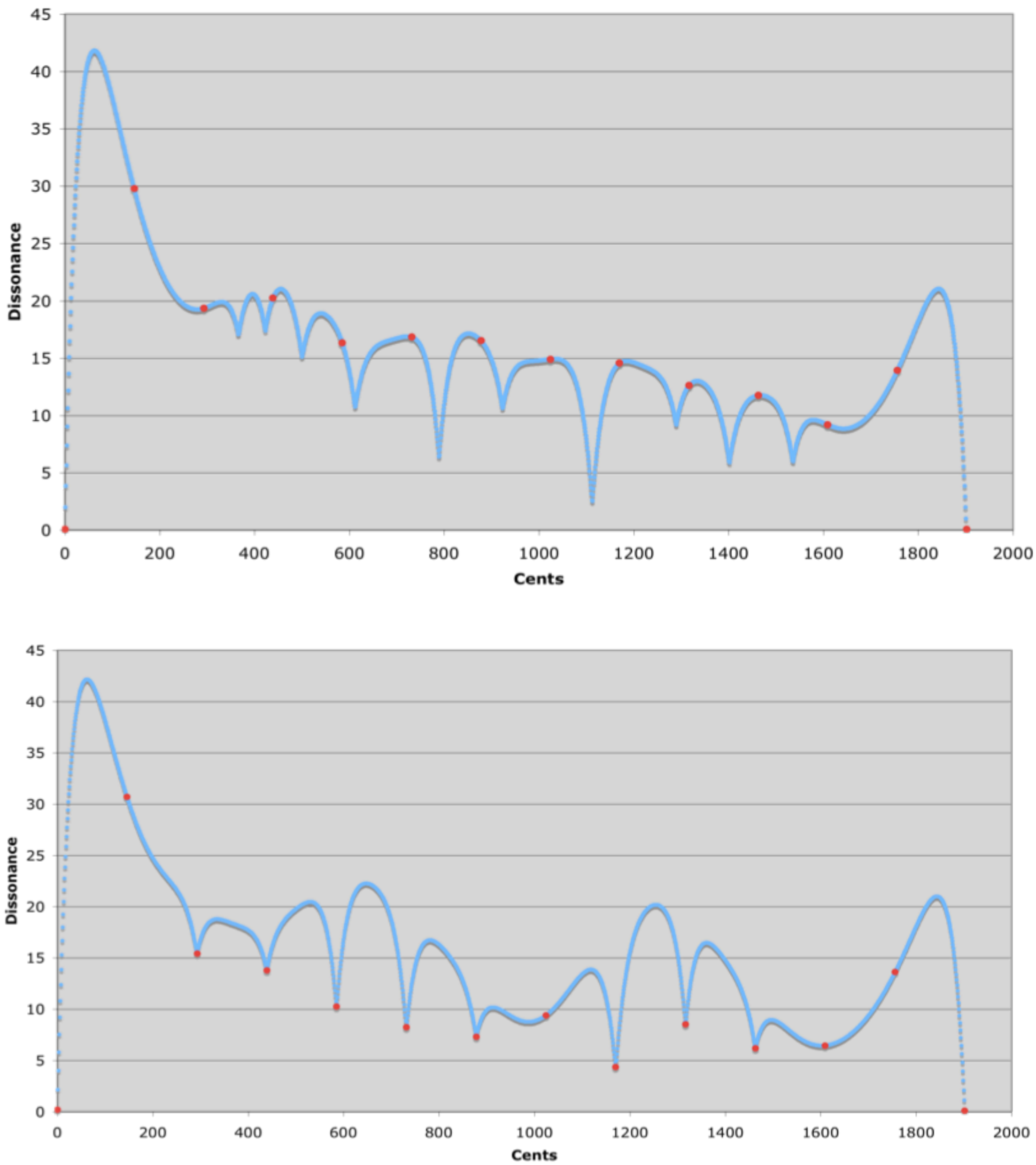

Fig. 23: The dissonance curve for a harmonic spectrum stretched by 3 reveals high sensory dissonance for the BP scale degrees (top) which can lessened by adjusting the partials to the nearest BP pitch (bottom).

Both spectra, the adjusted and non-adjusted ones, produce a fundamental pitch with a certain ambiguity as both the first and second partials are audible, the second being slightly more salient than the first, though, particularly in fast motion.

One striking effect of using stretched spectra with the BP triad was that root position and inversions sounded much similar to each other and therefore could more easily be identified as such.

Partial #	Stretched	Stretched & adj.	Partial #	Stretched	Stretched & adj.
1	48.9	48.9	13	2849.5	2824.3
2	146.7	146.7	14	3204.6	3073.3
3	278.9	288.4	15	3574.9	3639.3
4	440	440	16	3960	3960
5	626.7	617	17	4359.4	4309.4
6	836.7	865.1	18	4772.8	4689.5
7	1068.2	1024.4	19	5199.8	5103
8	1320	1320	20	5640.19	5553
9	1590.9	1563.1	21	6093.7	6042.7
10	1880	1851	22	6559.9	6575.6
11	2186.6	2191.8	23	7038.8	7155.4
12	2510	2595.4	24	7530	7786.4
Centroid	3071.6	3069.3			

Table 6: Frequencies for a stretched spectrum in comparison to its adjusted version for which the partials were bent to the nearest BP pitch. Both spectra have nearly the same centroid.

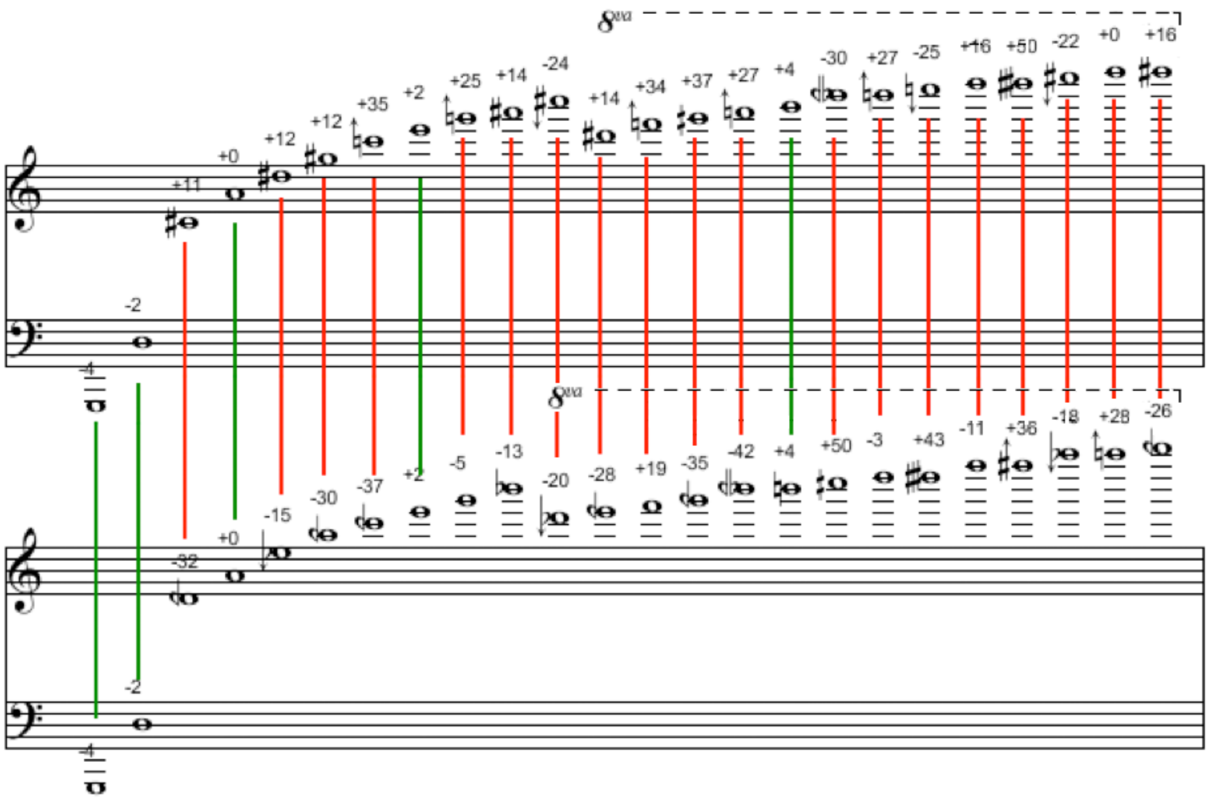

Fig. 24: Frequencies from Table 6 in eighth-tone notation. The red lines refer to partials adjusted to the BP scale (the original spectrum is on top), while the green lines indicate unchanged pitches corresponding to partial numbers 1, 2, 4, 8 and 16.

4 Ear training

In the winter semester 2013 a class in Bohlen-Pierce ear training was given by the authors at the Hamburg University of Music and Theater. The lessons served two purposes: on the one hand to enable music students to identify tones, intervals and chords of the scale and on the other hand to explore if it is possible to develop a new pedagogical and theoretical method of ear training specifically designed for the BP scale.

Different and mostly practical questions concerning note naming and notation had to be answered before we could start the training (see sections 2 and 3)

Once familiar with the notation and the note naming, the students learned the basics on the BP scale, its history, its construction as well as new terms such as the tritave. Musicians are used to reading scores in the five-line system even for microtonal music. In the beginning, both the conventional and new notation systems were presented so that the students could actually realize the difference between the 12-tone equal tempered (12edo) and the BP scale (see Fig. 6).

Afterwards we proceeded to the main ear training. Different exercises were presented starting by asking the students to sing the BP chromatic scale and get familiar with the intervals. In order to sing the scale at ease, the students were asked to sing the scale in the range they felt more comfortable. In other words, the exercise was adjusted to every person's voice range. The students did not have any previous practical experience with the BP scale, so in order to facilitate the exercise we recommended they divide the scale into 2 segments which form 2 diminished 7^{th} chords—similar to diminished tempered chords. The segments are pictured in the figure below (Fig. 24):

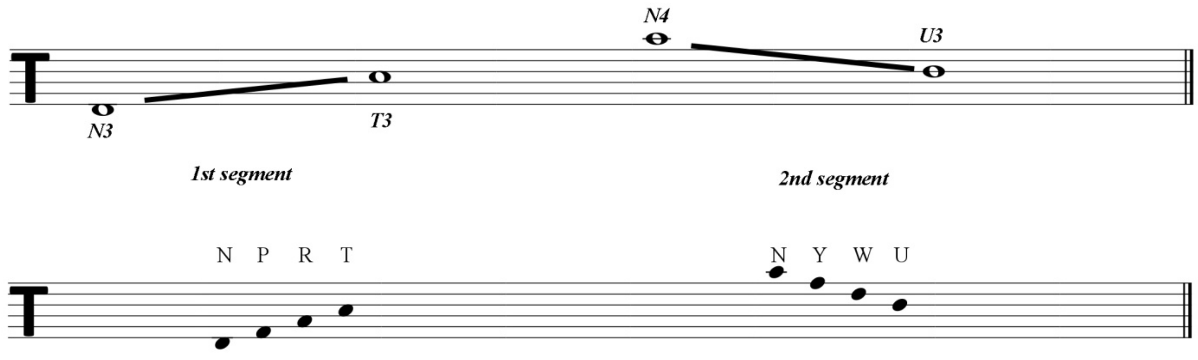

Fig. 24: BP chromatic scale divided into two main segments.

This way it was possible to think of successions of minor thirds for example between the notes N-P or R-T. The students had to build sequences to sing the step between the minor thirds. Once they had accomplished this, they put the two segments together and they were able to sing the whole BP scale.

We then proceeded by hearing and singing all intervals of BP scale in upward and downward movement. Some intervals of the BP scale are perceived as similar to the intervals of the tempered scale. More specific the BP 2^{nd} is very close to a minor third, the BP 4^{th} to a tritone, the BP 6^{th} to the major sixth, the BP 7^{th} to a minor seventh (Fig. 3) and the BP 10^{th} to a major tenth.

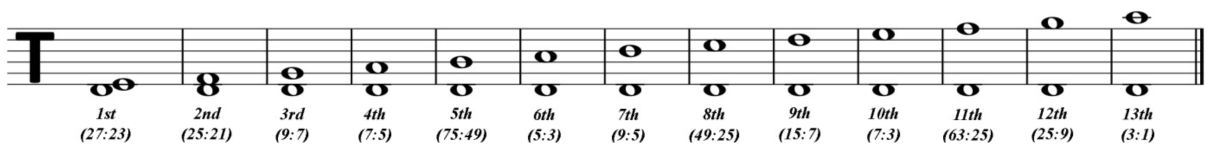

Fig. 25: Intervals and ratios of the BP scale

Difficulties were observed in two areas: firstly, the students, when listening to the BP intervals, compared them with those of the tempered scale and, secondly, they were confused when asked to name the intervals. For instance, the naming of intervals BP 3^{rd} to BP 10^{th} corresponds to familiar thirds, fourth, fifths etc., while BP 0^{th}, BP 1^{st}, BP 2^{nd}, BP 11^{th} and BP 12^{th} clearly does not. Particularly, calling a prime the BP 0^{th} and a second the BP 1^{st} was a bit of a mind twister.

By and by, though, the students were able to familiarize themselves with these peculiarities. It was also observed that the students could identify easier und more precise the BP intervals when the intervals were played on a keyboard with a sound similar to clarinet. In contrast, the sound of piano seemed to make the recognition of these intervals more difficult.

The next exercises of the training included dictations of short melodies. For these exercises the Lambda and the Pierce modes were preferred (Fig. 26).

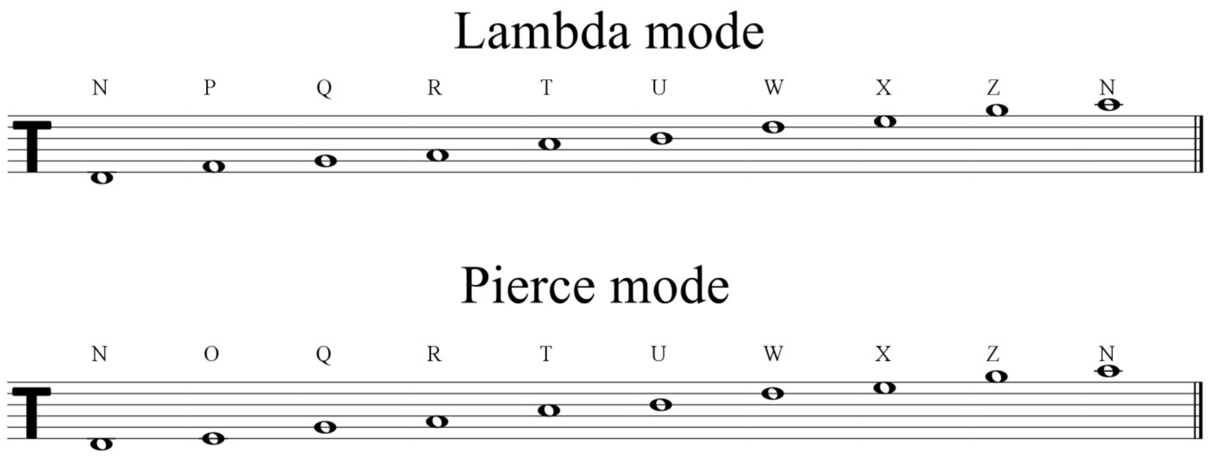

Fig. 26: The two modes used for the BP ear training: the lambda and the pierce mode.

The melodies in the beginning were simple by avoiding large leaps (with the exception of the tritave) and focusing on pitch and not on rhythm. Fig. 27 shows some examples.

Fig. 27: Three different dictations for the ear training course.

As the time passed by, the melodies got more complex in rhythmical and melodic variety and we finally enriched them by including a second voice (Fig. 28). Surprisingly, the students made minor errors.

Fig. 28: Dictations including two voices.

In the last lessons of the semester we concentrated also on BP chords. The main BP chord is built on the ratio 3:5:7 and is named the BP wide triad. Its intervals are the BP 6th (N-T) and the BP 4th (T-X). The chord has also two inversions: the first inversion (5:7:9) also called as narrow triad and the second inversion (7:9:15) as shown in the figure below (Fig. 29, also see section 3):

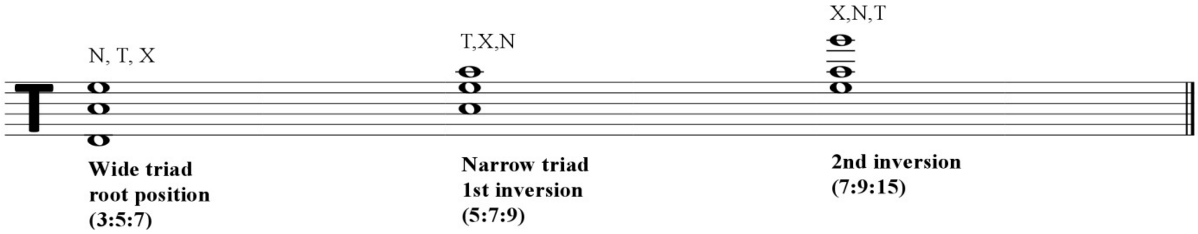

Fig. 29: The main BP chord and its inversions.

The students could distinguish between the wide and the narrow triad though they had some difficulties telling the narrow triad from the 2nd inversion and failed altogether at isolating and singing the middle note of a chord in either inversion.

At the end of the semester we concluded that the students were able to acquire the new scale as well as the new notation system. While in the beginning of the course the students needed an adjustment time to understand the new scale and to make the mental transition from the twelve-tone, octave-based system to the thirteen-tone tritave-based system, they were eventually capable of singing, notating and distinguishing intervals of the BP scale, despite the unfamiliar terrain[46]. During the course – in a way – both our auditory and visual perception has to be calibrated in order to be able to understand the tuning and that seems to function.

The first step is done. We have laid the foundation for a new approach to learning the BP scale and thus making it more approachable to musicians. However, we still need much more work in order to accomplish a complete theoretical and practical learning method of the tuning, which eventually will help us to spread it amongst musicians as an alternative source of inspiration.

5 Experiments

Introduction

Our brain has the ability to identify grammatical and hierarchical structures in music such as musical harmony[47], timbre[48], consonance[49] or contour discrimination[50].

46 American singer Maureen Chowning who has worked with BP composers such as Richard Boulanger has mentioned in a personal communication that picking the scale by ear didn't pose any difficulties for her.

47 C.L. Krumhansl and E.J. Kessler: Tracing the Dynamic Changes in Perceived Tonal Organization in a Spatial Representation of Musical Keys. *Psychological Review* 89 (1983): 334-368.

48 B. Tillmann, E. Bigand, N. Escoffier and P. Lalitte: The influence of musical relatedness on

Musical grammar facilitates the understanding of a particular music idiom even if the idiom is unfamiliar to the person[51]. Musicians mostly learn music in tonal contexts and specific tonal hierarchies. Krumhansl and Shepard[52] introduced the probe-tone method to demonstrate how people evaluate notes when presented in a diatonic context. Their findings showed that (1) frequency distance between tones plays an important role in the preference of the note, (2) intervals separated by an octave are closely related, (3) preferable tones are the tones near the tonal centre, and (4) preference of notes is associated with hierarchical patterns and more specific most preferred notes are the notes of the tonic triad (the 3rd and 5th note of the scale).

Loui[53] conducted a series of experiments in order to investigate if acquisition of artificial musical grammar is possible based on the BP chromatic scale. The experiments consisted of different phases including probe-tone ratings, melody recognition after exposure, and preference ratings. It was found that (1) participants could recognize melodies (in BP scale) they heard before, (2) probe-tones ratings correlated to the frequency of exposure, and (3) melodies that the participants were exposed to were preferred more than new melodies. Furthermore it was shown that timbre and musical transposition did not affect melody recognition. People were able to learn the new system after exposure regardless of timbre and harmony.

During the course we observed that students were slowly able to adjust to the new system. We noticed that the students on the one hand could recognize different BP modes, especially the Lambda and the Pierce modes, on the other hand the students would state that clarinet timbre seemed more appropriate than, for instance, piano timbre. Therefore, two main questions were posed: (1) can timbre influence concordance judgments of BP intervals, and (2) are tonal hierarchies as proposed by Krumhansl and Shepard applicable to the BP context? In order to answer these questions, two experiments were conducted. The first experiment used the probe-tone method while the second experiment focused on concordance ratings for BP intervals.

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timbre discrimination. *European Journal of Cognitive Psychology,* 18 (2006): 343-458.

49 J.J. Bharucha and K. Stoeckig: Reaction time and musical expectancy: priming of chords. *Journal of Experimental Psychology: Human Perception & Performance,* 12 (1986): 403-410.

50 P. Loui, and D. L. Wessel: Harmonic Expectation and Affect in Western Music: Effects of Attention and Training. *Perception and Psychophysics,* 69 (2007): 1084-1092.

51 F. Lerdahl and R. Jackendoff: *A generative theory of tonal music*. Cambridge, MA: MIT Press, 1983.

52 C.L. Krumhansl and R.N. Shepard: Quantification of the Hierarchy of Tonal Functions Within a Diatonic Context. Journal of Experiment Psychology: Human Perception and Performance, 5 (1979): 579-594.

53 Psyche Loui: *Acquiring a New Musical System*. PhD thesis, 2007. Source: http://cnmat.berkeley.edu/library/acquiring_new_musical_system (accessed June 01, 2014).

Probe-Tone Experiment

Participants

7 subjects (male: 4, female: 3; mean age: 26.85) participated in the experiment. The subjects were all students of the Hamburg University of Music and Theater. All participants undertook a BP ear training course for 4 months.

Methods and Materials

The experiment was conducted in two phases. The first phase took place during the first session of the ear training class and the second phase took place after four months of weekly ear training. The procedure of the experiment was exactly the same for both phases.

Two BP modes were used: Pierce and Walker B. Both modes consist of 9 notes, though different steps build the scales. The Pierce mode is based on the steps 1-2-1-2-1-2-1-2-1 whereas the Walker B mode is based on the steps 1-2-1-1-2-1-2-1-2. The reason for using these two modes lay in their differing interval content: While Pierce contains the tonic triad and leaps over the wolf tones, Walker B contains both wolf tones and instead misses the stable BP 6^{th} which in BP assumes a role akin to the perfect fifth in 12edo. We therefore expected (and ultimately got) a flatter tonality profile for Walker B.

The participants listened to a series of ascending tones of either the Pierce mode or the Walker B mode save the last tone (Fig. 30). Then, after a short interval a probe tone was presented. The participants were asked to judge how well the tone completed the scale on a numerical scale (1 = very bad, 7 = very good). All 13 notes of the chromatic BP scale were presented as final tones (Fig. 31).

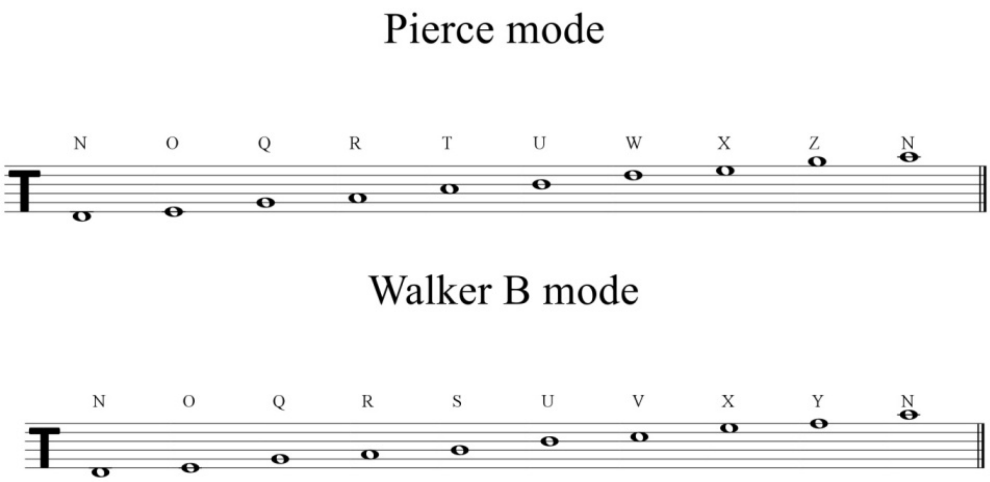


Fig. 30: The Pierce and Walker B mode based on the new notation.

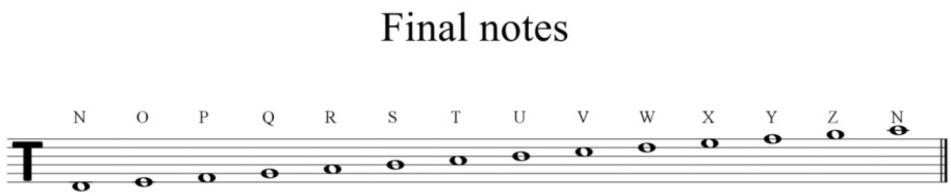


Fig. 31: Final notes to complete the sequence. All notes of the BP scale were presented in random order. N3 and N4 actually create the same sensation as we employed a circular Shepard-like tone.

Preparing for the experiments, we observed that when creating a Shepard-tone-like test tone with tritaves only, the resulting sound did not fuse satisfactorily and created an ambiguous pitch sensation in the listener. This effect could be somewhat lessened using a triangle wave instead. We therefore used tones synthesized with 6 triangle oscillators (and a spectral envelope in the shape of a bell curve with an average centroid of 742.5 Hz, SD = 39). The modes as well as the probe tones were presented in random order. The participants heard the samples through headphones.

Some remarks on statistics & results

Statistics is useful for analyzing, evaluating and presenting collected data from surveys or experiments. Different statistical methods are used to analyze data which imply different mathematical models. In experimental research we observe the reactions of people when we change or manipulate something. For example, in the experiment mentioned above, we present 2 BP modes with different final notes and we want to find (1) how people evaluate these final notes, and (2) if the ear training plays a role in this evaluation. People, who participate in experiments, represent a sample. When we collect the answers given by our sample, we can observe a central tendency (if the final note x was evaluated as very good, good or very bad). This central tendency is called also mean. Because we have two conditions in these experiments (evaluation before ear training / evaluation after ear training), we have also different means. In this experiment we use the t-test. In simple words a t-test compares the various means and actually tells us if the means are different from each other. Normally we expect that there are no differences between the means. For example, we expect that there are no differences between the evaluation of a final tone before or after training. However, if there is indeed a significant difference before and after training, this suggests that ear training actually affected how people evaluated the final note.

Here we have the situation that a final note was twice presented in the same context: before and after training. That means that we have two means for every final note. Therefore we use a paired-samples t-test which compares the means of these two conditions for a single sample (for the same group of participants).

Paired-samples t-tests were conducted in order to compare the means between the two conditions: before and after the ear training. In Pierce mode the participants evaluated the second note of the mode more positively after the training (M = 5.00, SE = 0.577) than before the training (M = 3.71, SE = 0.286); t (6) = -2.714, p < .05, r = .74.

Specifically the note V (8), which does not belong to the notes of the scale, was evaluated as the least suitable to complete the scale, whereas the note Z (12) as the most suitable to complete the scale in both tests. It was observed that after ear training the participants evaluated the notes O (1) and Y (11) more positively than before. The opposite was observed for the notes P (2) and T (6) (Fig. 32). Additionally, the note U (7) was rated more positively in both conditions than the note T (6).

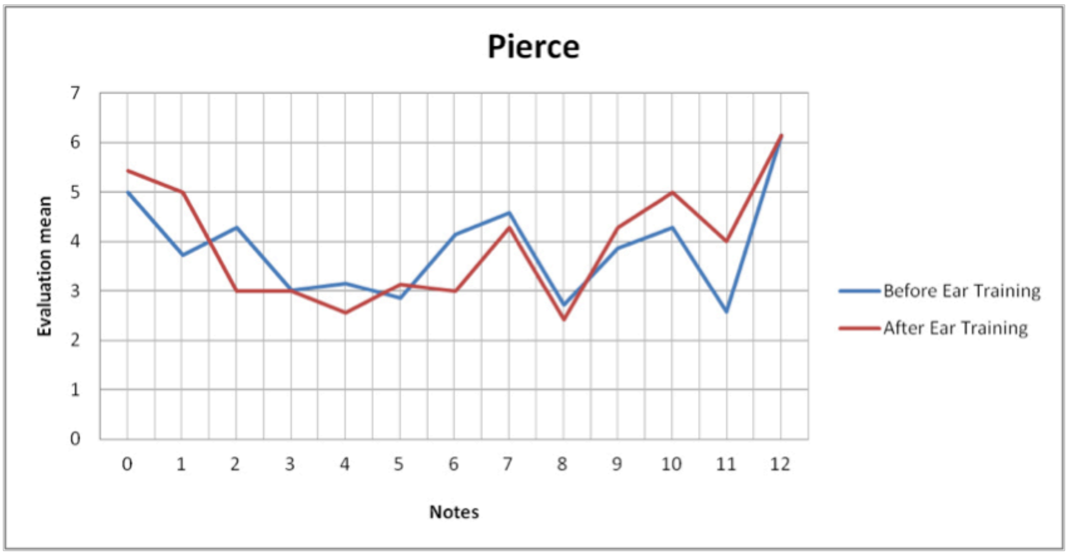


Fig. 32: Ratings given for the Pierce mode before and after the ear training.

No differences were observed from the t-tests between the two conditions for the Walker B mode. Specifically the note O (1) was evaluated as the least suitable note to complete the scale, whereas the note Z (12) as the most suitable note to complete the scale in both conditions though the note does not belong to the scale. It was observed that after the ear training the participants evaluated the note T (6) more positively than before ear training. The opposite was observed for the note N (0) (Fig. 33).

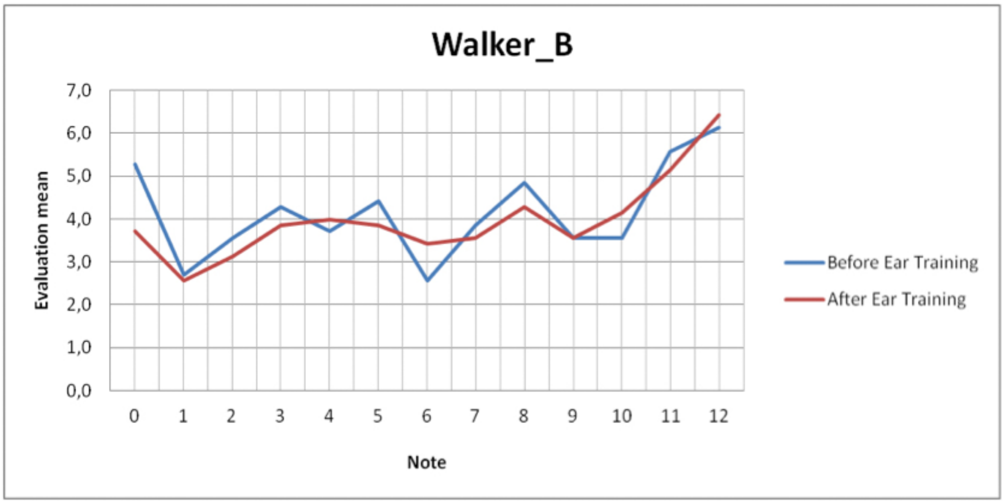


Fig. 33: Ratings given for the Walker B mode before and after ear training.

The participants rated higher the penultimate tone of the scale, when the tone was presented as a probe tone. This can be attributed to a reinforcement effect, particularly if the tonal context is not strong enough to keep the fundamental in the memory. We can only speculate why T, being the second tone of the wide triad on N, fell out of favor after ear training and are tempted to attribute this to unanticipated linear effects, possibly to a descending BP 5^{th} from Z down to U, which despite its oversize is probably still acceptable for two notes in succession.

Similarly to the findings of Krumhansl and Shepard the results above demonstrate that notes that belong to the scale, are more preferred than notes that do not belong to the scale. For the reasons mentioned before, this was more evident in the Pierce mode than the Walker B mode (Fig. 34).

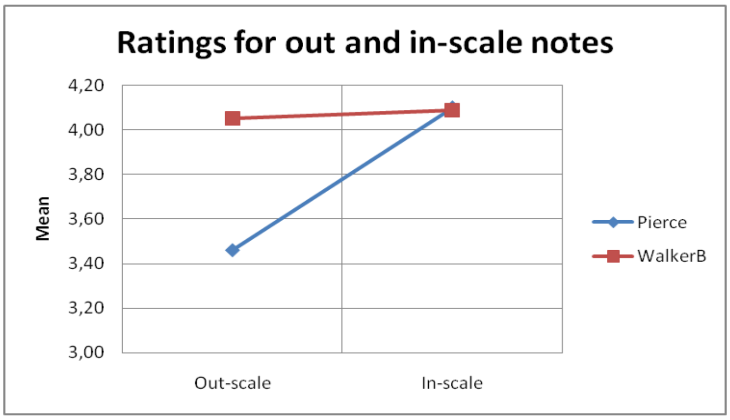


Fig. 34: Ratings for notes included in the scale and for notes not included in the scale.

Experiment on Intervals

Participants: 6 subjects (male: 4, female: 2; mean age: 36.2) participated in the experiment. The subjects were students of the Hamburg University of Music and Theater and professional musicians (mean of music education: 24.2 years).

Methods and Materials

The participants listened to intervals (simultaneous tones) of the BP chromatic scale. The intervals were played with two synthesized sounds: one resembling piano and another resembling clarinet. The sounds were selected from the instrument bank of the Max based program MaxScore. The intervals were presented in a random order and repeated twice. The participants were asked to judge if the intervals were concordant or not by selecting a number of a numerical scale (1 = discordant, 5 = concordant). The sounds were diffused by two loudspeakers placed in the room.

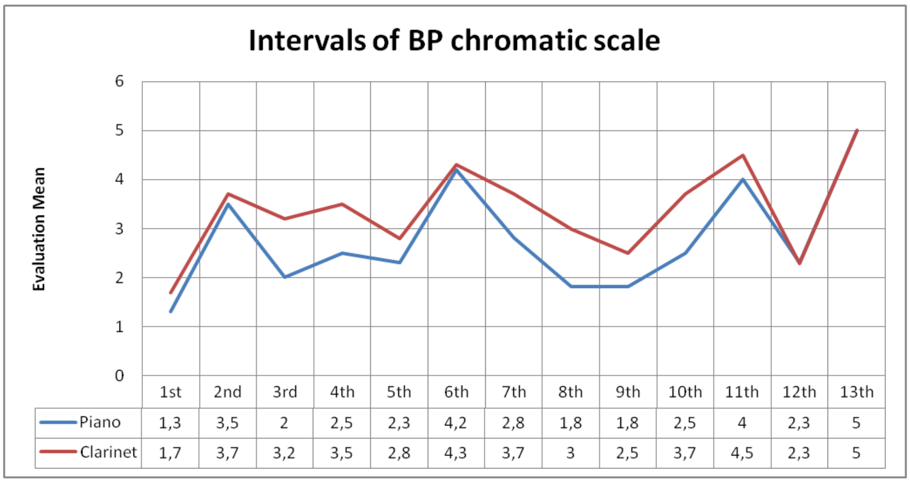
Results

As shown in the table below (Table 7) the participants judged the BP 1^{st} as the most discordant interval (1.50) while they judged the tritave (BP 13^{th}) as the most concordant interval (5.00). The BP 5th, 8th and 9th were also evaluated as mostly discordant. The BP 6th and 11th were mostly perceived as concordant intervals.

BP Interval	Mean
1st	1.50
2nd	3.60
3rd	2.60
4th	3.00
5th	2.55
6th	4.25
7th	3.25
8th	2.40
9th	2.15
10th	3.10
11th	4.25
12th	2.30
13th	5.00

Table 7: Ratings given for the different intervals (1 = discordant, 5 = concordant)

Paired-samples t-tests were conducted in order to compare the means between two conditions: intervals presented with piano timbre and intervals played with clarinet timbre. The participants evaluated the BP 4th more positively when presented with clarinet timbre (M = 3.50, SE = 0.428) than when presented with piano timbre (M = 2.50, SE = 0.342); t (5) = -3.873, p < .05, r = .87. The results show that the timbre of the sound influenced the way the participants judged the intervals. Intervals such as the BP 3rd, 4th and 8th, which were evaluated in general as discordant, were perceived as more harmonic when presented with clarinet timbre (Fig. 13). As stated before, intervals in BP are based on odd frequency ratios with factors 3, 5, and 7. In the low, chalumeau, register of the clarinet, odd harmonics are more prominent than even harmonics and that would explain why the intervals with clarinet timbre were perceived as more concordant.



	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th
Piano	1,3	3,5	2	2,5	2,3	4,2	2,8	1,8	1,8	2,5	4	2,3	5
Clarinet	1,7	3,7	3,2	3,5	2,8	4,3	3,7	3	2,5	3,7	4,5	2,3	5

Fig. 35: Ratings given for different intervals when presented in piano and clarinet sound.

Discussion

The ratings obtained in experiment 1 clearly showed that via ear training, participants were able to adjust to and learn a new, previously unknown system. This supports the idea that our brain is capable, after a specific period of exposure, to identify grammatical and hierarchical structures even if the musical system is unknown.

Two results were surprising and may be attributed to the following two gestalt effects: proximity and similarity. Firstly, the antepenultimate and penultimate tones of the BP scale received high ratings when presented as probe tones (proximity), and secondly, BP 7th was preferred over BP 6th as the former can be reached by leaping down the interval of a familiar, albeit sharp, just fifth (similarity). However, the last effect was only observed in the Pierce mode. To gain more insights, this experiment would have to be repeated with a) a descending scale and b) cadential progression. The data somehow also suggest that while presenting the scale during the experiment the fundamental looses its grip on the listeners and that more training and practice is needed until the BP modes and their tonal hierarchies are fully internalized.

The results of experiment 2 are twofold:

Firstly, it showed that the timbre of an instrument does influence the way we judge BP intervals. It seems that BP scale sounds “correct” or almost “right” when we hear it played by a clarinet. Students of the ear training course often complained that the scale was sounding “wrong” when played by another instrument but the clarinet. However, the curves show the same tendencies for both timbres. The sensory dissonance calculations (*Fig. 9*) suggest that while the clarinet spectrum has a smoothing effect on the unevenness of certain BP intervals (such as the wolf tones), the ratings are ultimately based on other phenomena (such as internalized harmonic relationships or gestalt principles).

Loui[54] also showed that timbre did not affect melody recognition or ratings for

54 ibid.

familiar melodies in a BP grammar though the study did not focus on instrumental timbres as much as on manipulation of spectral characteristics using tritave-based Shepard tones.

Secondly, preference ratings for the intervals of the BP chromatic scales seem to be generally related to sensory dissonance and harmonic consonance supporting the theory that spectrum and tuning are strongly related. Comparing the curves in Fig. 36 we notice similar tendencies for most intervals, particularly BP 1st, BP 5th, BP 6th, BP 7th, BP 8th, BP 10th, BP 12th and (of course) BP 13th.

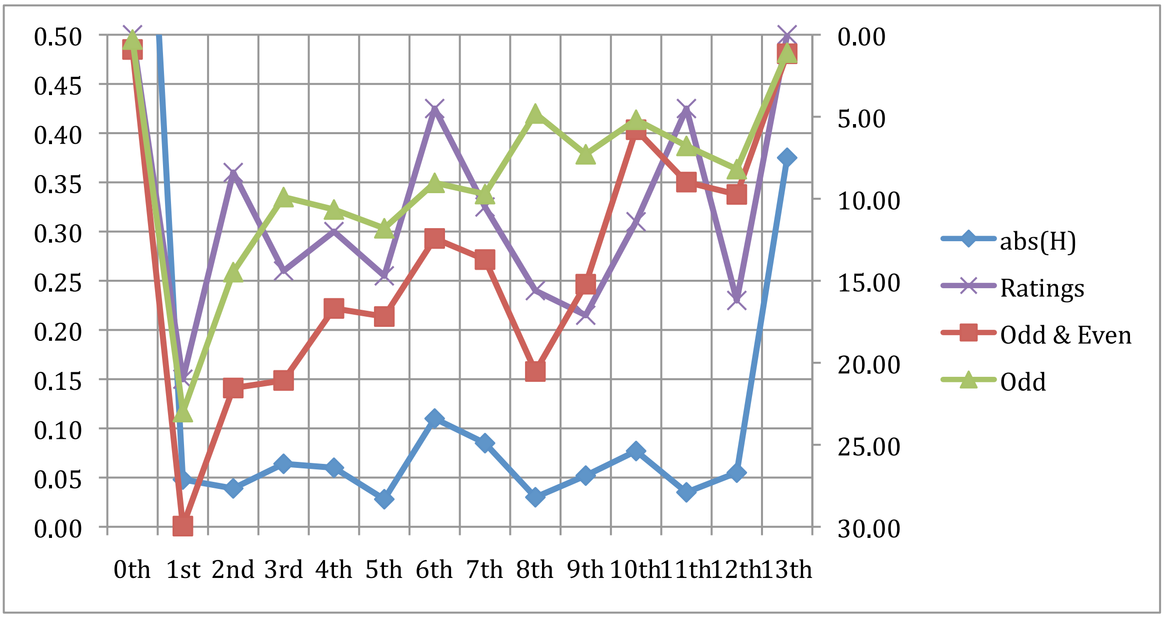


Fig. 36: Comparison between the values for sensory dissonance for even and odd harmonics (red), as well as just odd harmonics (green), harmonic consonance (blue) and averaged subjective preference rating for the BP intervals (the scale on the left refers to harmonic consonance and preference rating, whereas the reversed scale on the right refers to dissonance calculations).

However, the table also shows that the BP 2nd and the BP 11th strongly differ from the values expected. Both intervals are close to tempered intervals; the minor 3rd and the major 10th. This indicates that the learned relationships of the 12-tone tuning still greatly influence the perception of the BP scale, at least in the isolated context of the experiment. It would be interesting to see whether continued exposure to melodies or entire pieces with a particular BP grammar change the ratings in any way.

We conclude that the non-octave Bohlen-Pierce scale can be learned and deserves to be explored as an optional musical system for further artistic inspiration. However, many more experiments have to be conducted in order to understand what perceptual and conceptual processes are involved when we hear this new and unusual musical system.

6 Bohlen-Pierce Instruments

Interest in BP music has grown over the past years, and a number of instruments have been developed for the scale. At present, besides digital keyboards, there are BP instruments such as guitars, clarinets, and pitched percussion, some of which shall be discussed in the following.

Pitched Percussion Instruments in Bohlen-Pierce

Building pitched percussion instruments in BP is possible in many ways, and often percussionists make their own BP instruments. Chimes and metallophones have been made so far[55]; a BP xylophone or BP marimba would be a good idea, too. In marimba building it is common practice to shape the bars in a way that the 2nd and / or 3rd partials are suppressed[56,57]. The absence of the 2nd partial smoothes the sound of the BP scale by avoiding dissonance between the 49:25 BP interval and the harmonic partial 2:1 which is found in many instruments' spectra.

Thinking several steps further towards a big orchestra setup, BP timpani, gongs, or crotales would be an option, too.

In the past years digital percussion instruments have come to the market, some of which are tunable. Retuning a digital drum to BP is a relatively easy and inexpensive way to enrich a Bohlen-Pierce band setup[58].

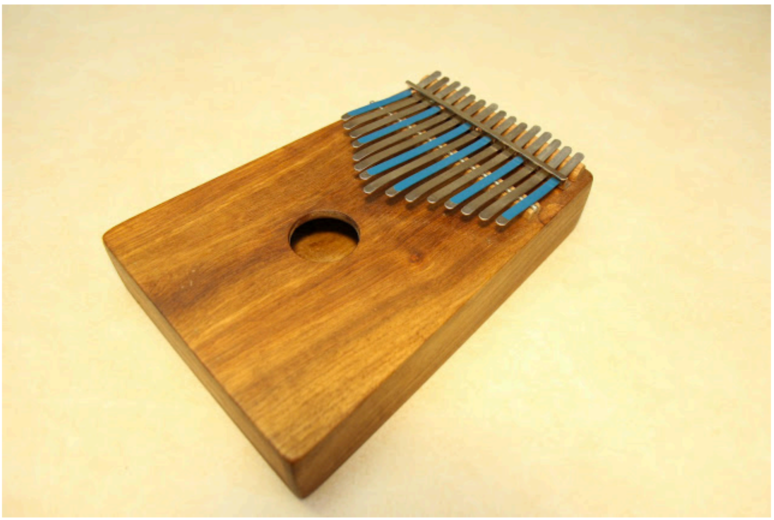


Fig. 37: A Hugh-Tracy alto kalimba has enough tines to represent an entire tritave.

55 http://transpectra.org/instruments.html (accessed June 01, 2014).

56 Ingolf Bork et. al., Comparison between Modal Analysis and Finite Element Modelling of a Marimba Bar, Braunschweig / Wolfsburg / Toulouse, 1997.

57 M. Cambell and C. Greated: *The Musician's Guide to Acoustics*, Oxford: Oxford University Press, 1994.

58 http://bp.b0b.com/2012/12/handsonic-hpd-15 shows how to retune a Roland HandSonic HPD-15 digital drum (accessed June 01, 2014).

String Instruments in Bohlen-Pierce

It is possible to play the BP scale on unfretted string instruments. Ensemble tranSpectra gives practical advice about retuning the chords of bowed string instruments on their website[59]. However, different tuning solutions may be possible, too. In general, BP string playing is not as common yet among string players, and thus there still is a lack of practical experience.

Bohlen-Pierce Guitars

BP guitars are made by Ron Sword and Jean-Pierre Poulin, among others. Sword refrets and retunes acoustic guitars to BP and has an electric guitar model as well, the 9-string Bohlen-Pierce touchstick / guitar. A variation of Sword's acoustic models is the triple BP guitar which subdivides every BP step into three micro-steps after a scale proposed by Paul Erlich[60].

Fig. 38: Ron Sword and his 9-string BP touch stick/guitar

Besides refretted acoustic guitars, Jean-Pierre Poulin builds electric guitars and

59 http://www.transpectra.org/instruments.html (accessed June 01, 2014).
60 Ron Sword: *Bohlen-Pierce Scales for Guitar,* IAAA Press, 2009.

basses in BP.

An anonymous private person refretted and retuned a pedal steel guitar to BP[61].

Bohlen-Pierce Clarinets

The clarinet is the most obvious choice for a BP woodwind instrument. It overblows at the perfect twelfth (tritave) and in the low and middle registers the 2nd and 4th partials – octave and double-octave – are suppressed while the third partial – the twelfth – is predominant; for this reason it fits the BP scale in a natural way.[62]

BP clarinets are currently made by Stephen Fox, Toronto[63] and were first used in public concerts in 2008. Fox chose a' = 440-442 Hz as the reference pitch for the BP soprano clarinet which at the same time set the standard for all other BP instruments. From the clarinet maker's and clarinetist's point of view this choice of reference pitch is handy because a' results from overblowing the entire tube length of the regular B♭ clarinet. For this reason, the instrument has the same body length with a sounding d = 147 Hz as the bottom note, and the same mouthpiece can be used. Fox chose the same key arrangement as for the Boehm clarinet, i.e. four keys for the right little finger and three keys for the left one (right: keys e/b', f/c', f#/c#' and g#/d#' of the traditional clarinet; left: e/b', f/c' and f#/c#'). The remainder of the tritave is produced by six open finger holes, a thumb hole and a key above the first finger hole, operated by the left index finger, equal to the traditional throat a' key.

Not as common as the BP soprano clarinet is the BP tenor clarinet. It is pitched six BP steps below the soprano clarinet and is about halfway in size between the traditional alto and bass clarinets. Fox based its design on an alto clarinet with the same mouthpiece, neck and bell. It uses the same fingering pattern as a soprano clarinet up to the top register.

Fox has also made a BP clarinet of the size of the traditional E♭ clarinet. A BP contra clarinet has not been built yet.

Fig. 39: A BP soprano clarinet made by Stephen Fox in Toronto

61 http://bp.b0b.com/2012/11/sierra-pedal-steel (accessed June 01, 2014).
62 Jürgen Meyer: *Acoustics and the Performance of Music*, New York: Springer, 2010.
63 http://www.sfoxclarinets.com/bpclar.html (accessed June 01, 2014).

Bohlen-Pierce Pan Flutes

The pan flute is another woodwind instrument whose spectrum is mainly made up of odd harmonics. A BP pan flute has been made in 2009 by Ulrich Herkenhoff. It has 27 bamboo pipes covering a pitch range from 265 Hz (c' +22¢) to 2385 Hz, two tritaves above the flute's lowest note.

Fig. 40: The one and only BP pan flute made by Ulrich Herkenhoff

Other Woodwind Instruments in Bohlen-Pierce

Is it possible to build other woodwind instruments in BP, such as flutes, saxophones, oboes and bassoons? These instruments all overblow at the octave. In the BP scale, the interval nearest to the octave (BP eighth) is 30¢ smaller than a perfect octave, which is a conceptual problem with woodwind instruments other than the clarinet or pan flute. Theoretically, this problem can be avoided by using differing fingerings in the lower and upper register. Practically, this may cause problems to the player, and it may result in a big difference in sound quality between registers. To use the same fingerings in the upper and lower register would make a change of design necessary, changing the bore dimensions for instance; by this an instrument could be made that has a flat octave, or rather a "non-octave". If this were physically possible, it would possibly make the low register unstable, especially with conical bore reed instruments. Since no experiments have been made in this direction yet, it is not possible to predict with certainty[64].

In 2010, Fox unveiled the first Bohlen-Pierce alto recorder. It was constructed by plugging holes of a normal alto recorder and re-drilling them in new places. The reference pitch is a', as with BP clarinets. The instrument still overblows at the octave which makes it necessary to find fingerings for the upper register in accordance to the required "non-octave".

*For further materials on the Bohlen-Pierce scale (including samples and audio recordings) please refer to p. 5 for a link to the **1001microtones** online repository.*

64 This information was given by Stephen Fox in his lecture during the first Bohlen-Pierce symposium in Boston, MA in March 2010. A recording of the lecture can be found on http://recordings.bcdixon.com/10-03-07_BP/ (accessed March 21, 2014).